A Wave Hop Propagation Program for an Anistropic Ionosphere

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FOREWORD

This work was performed for the Defense Atomic Support Agency, under the direction of Mr. Dow E. Evelyn, HQ, DASA.
A WAVE HOP PROPAGATION PROGRAM FOR AN ANISOTROPIC IONOSPHERE

Leslie A. Berry and J. E. Herman

This report documents a digital computer program for computing the propagation of LF and VLF radio waves using the wave hop theory. It supersedes NBS Report 8889. The documentation includes a description of the physical model, detailed mathematical formulas, a main function flow chart, a listing of the FORTRAN source deck, and the input and output of a sample case.

1. INTRODUCTION

This report documents a computer program for the full wave calculation of LF and VLF radio propagation under and anisotropic ionosphere using the wave hop theory. It supersedes NBS Report 8889, "FORTRAN Programs for Full-Wave Calculation of LF and VLF Radio Propagation" (Berry and Chrisman, 1965a). The revision is necessary because the original program was based on a formulation that is incorrect for an anisotropic ionosphere (see Berry, et al. (1969) for details). The error was significant if the magnitudes of the ionospheric polarization coupling (conversion) coefficients were comparable to those of the reflection coefficients. The program and all its subroutines have been completely rewritten in a more modern version of FORTRAN (CDC FORTRAN 63) and a number of improvements suggested by users have been incorporated.

The next section explains the concepts and model used, so that you will know when it is appropriate to use the program, how to determine the model, and what input information you must provide. Section 3 contains detailed mathematical formulas used, and section 4 describes the computer implementation. This last section includes a test case and a listing of the FORTRAN source deck.

An ionospheric reflection coefficient program which can be used to generate input data for the propagation program is described in the appendix.
2. THE PHYSICAL PROBLEM AND THE MATHEMATICAL MODEL

The problem solved by the computer program can be stated as follows:

**Given:** A ground-based source of vertically polarized radio waves\(^1\) of known frequency in the LF-VLF band, the electrical constants of the ground and the reflection characteristics of the ionosphere along a great circle path.

**Calculate:** The amplitude and phase of the propagated vertical electric field on the earth's surface as a function of distance along the path.

The geometry of the assumed model is shown in figure 1. The earth is a smooth sphere with radius a and electrical constants \(\sigma\) (conductivity) and \(\varepsilon\) (dielectric constant). The ionosphere is concentric with known reflection properties characterized by the ionospheric reflection coefficient matrix \(T(\psi)\), where \(\psi\) is the angle of incidence of a wave on the ionosphere. The height of the ionosphere above the earth, \(h\), can be thought of as the virtual, or phase, height. Distance, \(d\), is measured along the ground; we also use the angular distance \(\theta = d/a\).

We compute the vertical (radial) electric field at \(O\) due to a vertically polarized source at \(S\). The field is the vector sum of several components which travel to \(O\) via different paths:

\[
E(d) = \sum_{j=0}^{J} E_j(d),
\]

(1)

where the subscript \(j\) counts the number of times the energy has been reflected from the ionosphere. Thus, \(E_0\) is the ground wave that travels directly along the surface of the earth from \(S\) to \(O\). \(E_1\) is the "first hop" (shown as the solid line in figure 1) which has reflected once from the ionosphere; \(E_2\) is the "second hop" (shown as the dashed line in figure 1) which has reflected once from the ionosphere.

\(^1\) Lewis (1970) derives wave hop formulas for an elevated, arbitrarily-oriented dipole.
Figure 1. Geometry for wave hop propagation model.
figure 1); and so forth. The angle of incidence of the path of the \( j \)-th hop on the ground is denoted \( \tau_j \); the angle of incidence on the ionosphere is denoted \( \omega_j \).

The theory is developed by Wait (1961), Berry (1964), and Berry et al. (1969), and the propagation characteristics of the wave hops are discussed by Berry (1964, 1967) and Johler (1970). Wave hop theory is most appropriate at LF (30-300 kHz) for distances less than, say, 6000 km. At lower frequencies a VLF waveguide mode theory (Wait, 1968 and references cited therein; Pappert, 1968) program is usually more appropriate, especially for long paths, although the wave hop program will compute fields correctly down to a few kilohertz and out to distances beyond 10,000 km. This conclusion is illustrated by Morfitt and Halley (1970) who compare fields calculated using this wave hop theory with fields calculated using the NEL mode theory program (Sheddy et al., 1968).

To use the program, you must specify:

(a) The transmitter's operating frequency and effective radiated power.

(b) The ground conductivity and relative dielectric constant. These are assumed to be constant along the path. If the conductivity varies significantly along the path, the geometric mean of the path conductivities is a useful approximation. Morgan (1968) provides a worldwide VLF effective-conductivity map which is also useful at low LF. At high LF and above, conductivity maps prepared for MF frequencies, such as that prepared by Barghausen, et al. (1966) may be more appropriate.

(c) Ionospheric reflection coefficients as a function of angle of incidence. The coefficients should be given for the
entire range of angles of incidence that will be encountered by the program during execution. Belrose (1968), Watt (1967), and Berry and Chrisman (1965b) show graphs of the angle of incidence for various distances and reflection heights. The appendix of this report contains a reflection coefficient program that can be used to compute reflection coefficients for any given ionospheric profile, or reflection coefficients from different sources can be used. Belrose (1968) shows empirically derived reflection coefficients for frequencies in the upper LF and MF bands.

(d) The distances of interest. These are given by a minimum distance, a distance increment, and a maximum distance.

(e) The number of hops to be computed and summed. The number may be different for each of five (or fewer) distance ranges, because fewer hops are necessary at short ranges. The number of hops necessary for the series in (1) to converge depends on the amplitude of the ionospheric reflection coefficients, the ground conductivity, and the frequency. The amplitude of the individual terms in (1) are printed out, so it is possible to determine, after the fact, if enough terms were used, but advance determination of the minimum number of hops necessary still requires judgement. For daytime ionospheric reflection coefficients a crude rule-of-thumb is: number of hops = (distance in Mm) + 1. Fewer hops are needed at high LF, more are needed at night.
3. DETAILS OF THE MATHEMATICAL SOLUTION

3.1 Notation

\[ f = \text{frequency, Hz} \]
\[ P_r = \text{effective radiated power, watts} \]
\[ d = \text{distance along the great circle path, m} \]
\[ a_1 = \text{earth's radius} \approx 6.36739 \times 10^6 \text{ m} \]
\[ h = \text{phase reference height of ionosphere (loosely, "reflection height")} \]
\[ c = \text{speed of light} \approx 2.997925 \times 10^8 \text{ m/s} \]
\[ \mu_0 = 4\pi \times 10^{-7} \]
\[ \sigma = \text{ground conductivity, mho/m} \]
\[ \varepsilon = \text{relative dielectric constant of ground} \]

\[ T_{ee}, T_{em}, T_{me}, T_{mm} \] are ionospheric reflection and conversion coefficients (Johler and Harper, 1962). The subscripts indicate which vector (electric or magnetic) is in the plane of incidence; the first subscript refers to the incident wave and the second to the reflected wave. The equivalence to a more common notation is: \[ T_{ee} \equiv R_{||} \], \[ T_{em} \equiv R_{\perp} \], \[ T_{me} \equiv R_{\perp} \], and \[ T_{mm} \equiv R_{||} \].

\( \psi = \text{the angle of incidence on the ionosphere} \) \hspace{1cm} (2)

\[ w = 2\pi f, \quad k = \omega / c \] \hspace{1cm} (3)

\[ k_2 = \sqrt{k^2 \varepsilon - i \mu_0 \omega \sigma} = k \sqrt{\varepsilon - i \frac{\mu_0 \omega \sigma}{\omega}} \] \hspace{1cm} (4)

\[ a_2 = a_1 + h \] \hspace{1cm} (5)

\[ v = (ka_1/2)^{\frac{3}{2}} \] \hspace{1cm} (6)

\[ x = v^2, \quad y = (kh/v) \] \hspace{1cm} (7)

\[ z = 1.25/v^2 \]
\[ q_0 = -iv\frac{k}{\kappa_2} \sqrt{1 - \left(\frac{k}{\kappa_2}\right)^2} \]  

(8)

\[ q_0 = -iv\frac{k_2}{k} \sqrt{1 - \left(\frac{k}{k_2}\right)^2} \]

\[ \theta = d/a_1, \; x = \nu \theta \]  

(9)

\[ F = 30 \sqrt{\frac{\pi PD}{120}}, \; G = \frac{e^{-ikd+i\pi/4}}{d} \sqrt{\frac{\theta x}{\pi \sin \theta}} \cdot \frac{F}{2}. \]  

(10)

3.2 The Ground-Wave

The first term of (1) is the ground wave, \( E_0 \). It is given by (compare Wait (1962), Fock (1964))

\[ E_0(d) = -4\pi G \sum_{S} (1 + zt) \left(1 + \frac{3 + i \cot \theta}{8ka_1 + vt_s}\right) \frac{e^{-ixt_s}}{t_s - q_0^2}. \]  

(11)

The \( t_s \) satisfy

\[ W_1'(t_s) - q_0 W_1(t_s) = 0, \]  

(12)

where \( W_1(t) \) is the Airy function of the first kind defined by Wait (1962), and the \( t_s \) are numbered in order of increasing magnitude of the imaginary part. The first two factors in the \( s \)-th term differ from unity by a few percent, at most, and are frequently neglected in ground wave calculations.

3.3 The Wave Hops

For \( j \geq 1 \) in (1), the wave hops are (Berry, et al., 1969)

\[ E_j = I_{j1} T_{ee}^j + \sum_{M=2}^{j} I_{jM} C_{jm}, \]  

(13)

where
\[
C_{\text{jm}} = \sum_{k=1}^{M-1} a_{jm,k} T_{ee}^{j+1-M-k} (T_{em} T_{me})^k T_{mm}^{M-1-k} \quad (14)
\]

Then
\[
a_{jm1} = j - M + 1 \quad (15)
\]

and
\[
a_{jm,k} = \frac{(j+2-M-k)(M-k)}{k(k-1)} a_{jm(k-1)} \quad \text{for } k \geq 2.
\]

Both \( I_{\text{jm}} \) and the \( T_{ik} \) are implicit functions of \( d \). The functions are made explicit below, beginning with the path integrals, \( I_{\text{jm}} \).

The basic formula is (Berry, et al., 1969)
\[
I_{\text{jm}} = G \int_{\Gamma} \frac{1}{1+zt} e^{-ixt} W_1(t) W_2(t) (1+R_e)^2 p^{-j-M} R_e^{-M} \ dt, \quad (16)
\]

where the ground reflection coefficients are
\[
R_1 = -\frac{W_2'(t)/W_2(t) - q}{W_1'(t)/W_1(t) - q}, \quad i = e \text{ or } m, \quad (17)
\]

and
\[
p = \frac{W_2(t) W_1(t-y)}{W_1(t) W_2(t-y)}. \quad (18)
\]

\( \Gamma \) runs from \( \infty \) to 0 and down into the third quadrant to \( \infty e^{-i2\pi/3} \). However, (16) is an approximation to an integral involving Hankel functions of complex order (Berry, 1964; Berry and Chrisman, 1965c), and the approximation is valid only if \( \cos \tau_1 \) is small (refer to figure 1).

3.3.1 Geometrical Optics

For short distances, \( \cos \tau_1 \) is not small so we use the geometrical optics formula:
where

\[ I_{2J} \approx -i \frac{e^{-ikD_j}}{D_j} B_j \sin^2 \tau_j (1 + \hat{R}_e)^2 \hat{R}_e^{j-M} R_e^{M-1}, \]  

(19)

and the convergence-divergence coefficient is

\[ B_j = \frac{a_2}{a_1} \sqrt{\frac{D_j \sin \tau_j \cos \varphi_j}{a_2 \sin \theta \cos \tau_j}}, \]  

(23)

The Fresnel ground reflection coefficients are

\[ \hat{R}_j = \frac{s + q_1}{s - q_1}, \]  

where \( s = iv \cos \tau_j \)  

(24)

3.3.2 Saddle Point Approximation.

The relationship between (19) and (16) was shown by Wait (1961). Following his derivation, change the contour \( \Gamma \) so that we integrate from -\( \infty \) to \( \infty \). Since we cross no singularities with this move, it only changes the sign of (16). Then, if \((-t) > 1 \),

\[ W_k(t) \approx (-t)^{\frac{1}{4}} \exp \left[ \left(-1\right)^k i \left( \frac{2}{3} (-t)^{\frac{3}{2}} + \frac{\pi}{4} \right) \right] \]

and

\[ W'_k \approx (-1)^{k-1} i (-t)^{\frac{1}{2}} W_k(t). \]

Substitution into (16) yields
\[ I_{ym} \approx -G \int_{-\infty}^{\infty} e^{-i\Omega(t)} (1 + zt) (-t)^{\frac{1}{2}} (1 + R_e)^{\frac{2}{2}} R_s^{j-M} R_m^{M-1} \, dt, \quad (26) \]

where

\[ \Omega(t) = xt - \frac{j}{3} (-t)^{\frac{3}{2}} + \frac{4}{3} j(y-t)^{\frac{3}{2}}, \quad (27) \]

and now

\[ R(t) \approx \frac{(-t)^{\frac{1}{2}} - iq_1}{(-t)^{\frac{1}{2}} + iq_1}. \]

The saddle point approximation to (26) is

\[ I_{ym} \approx -i F \frac{e^{-ikd}}{d} \sqrt{\frac{\theta}{\sin \theta}} (1 - z\alpha^2)(1 + \frac{x}{2j\alpha}) (1 + R_e)^{\frac{2}{2}} R_s^{j-M} R_m^{M-1}, \quad (28) \]

where

\[ \alpha = (-t_0)^{\frac{1}{2}} = \frac{4j y-x^2}{4j x}. \quad (29) \]

It can be shown (Wait, 1961) that, for \( \cos \tau_j << 1 \),

\[ (-t_0)^{\frac{1}{2}} \approx v \cos \tau_j, \]

\[ \Omega(t_0) \approx k(D_j - d), \]

\[ (1 + \frac{x}{2j\alpha})^{\frac{1}{2}} \approx B_j, \]

\[ (1 - z\alpha^2) \approx \sin^2 \tau_j, \]

and

\[ d \frac{\sin \theta}{\theta} \approx D_j. \]
so (19) and (28), and hence (16), are approximately equal for the conditions \((-t_0) > 1\) and \(\cos \tau_j < 1\). This small region is large enough to overlap (16) and (19).

3. 3. 3 Numerical Integration.

When \(\tau_j\) approaches \(\pi/2\), (16) must be integrated numerically. We use the Wronskian (Wait, 1962)

\[
W_1'(t) W_2(t) - W_2'(t) W_1(t) = 2i
\]

and (17) to derive

\[
(1 + R_e)^2 = \frac{-4}{W_2^2(t) (W_1(t) - q_e W_1(t))} ,
\]

and hence

\[
I_{jm} = G \int \Gamma (4)(-1)^j(1 + zt) e^{-i\alpha t} \left( \frac{W_1(t-y)}{W_2(t-y)} \right)^j \left( \frac{E_1(t, q_e)}{E_1(t, q_\pi)} \right)^{j-M} \left( \frac{E_2(t, q_\pi)}{E_1(t, q_\pi)} \right)^{M-1} dt ,
\]

where

\[
E_\kappa(t, q_\pi) = W_\kappa'(t) - q_\pi W_\kappa(t) .
\]

3. 3. 4 Residue Series.

The integrand in (31) has poles of order \(j-M+2\) wherever

\[
E_1(t, q_e) = 0 ,
\]

and poles of order \(M-1\) wherever

\[
E_1(t, q_\pi) = 0 .
\]

Zeroes for both functions exist inside the contour \(\Gamma\); they are the well-known solutions used for calculation of ground wave propagation over a spherical earth or diffraction by a finitely conducting sphere (Wait, 1962; Fock, 1965). The zeroes of \(W_\pi(t-y)\) are all in the upper half plane outside the contour \(\Gamma\). Thus, \(I_{jm}\) can be evaluated by summing a residue series:
We will first discuss methods for calculating residues of high order poles; then return to the problem of finding the pole locations $t_s$.

Assume that we have an integral
\[ I = \oint \frac{A(t)}{B(t)} \, dt, \] (34)

where $A(t)$ is analytic on and inside the contour and $B(t)$ has a zero of order $N$ at $t = t_0$ inside the contour. Expand $A(t)$, $B(t)$, and their ratio in Laurent series around $t_0$ and equate coefficients of like powers to obtain (see for example Kaplan (1952), pp. 564-565)

\[ b_0 \hat{a}_0 = a_0, \]
\[ b_1 \hat{a}_0 + b_0 \hat{a}_1 = a_1, \]
\[ b_2 \hat{a}_0 + b_1 \hat{a}_1 + b_0 \hat{a}_2 = a_2, \]
\[ \vdots \]
\[ b_{N-1} \hat{a}_0 + b_{N-2} \hat{a}_1 + \ldots + b_0 \hat{a}_{N-1} = a_{N-1}, \] (35)

where
\[ a_i = \frac{d^i}{dt^i} \left( \frac{A(t)}{B(t)} \right) \bigg|_{t=t_0}, \] (36)

and
\[ b_i = \frac{d^{i+N}}{dt^{i+N}} \left( \frac{B(t)}{t^{N}} \right) \bigg|_{t=t_0}, \]

are the coefficients of the Laurent series for $A(t)$ and $B(t)$ respectively, and the $\hat{a}$ are the coefficients of the series for $A/B$. By definition

\[ \text{Residue} (t_0) = \hat{a}_{N-1}. \] (37)
The set of equations (35) can be solved recursively:

\[ \tilde{a}_0 = \frac{a_0}{b_0}; \text{ and } \tilde{a}_z = \left( a_z - \sum_{\ell=0}^{z} b_{z-\ell} \tilde{a}_\ell \right) / b_0 . \]  

Comparing (34) and (31), we see that if

\[ E_1(t, q_m) = 0 ("q_m\ poles") , \]

\[ A(t) = 4(-1)^j (1 + zt) e^{-ixt} \left( \frac{W_1(t-y)}{W_2(t-y)} \right)^j \frac{E^{M-1}_2(t, q_m)}{E^{M-1}_1(t, q_n)} , \]  

(39)

and

\[ B(t) = (E_1(t, q_m))^{j-M+2} . \]

If \( E_1(t, q_m) = 0 \), ("\( q_m \) poles"), then \( N = M-1 \) (there are no \( q_m \) poles for \( M = 1 \)),

\[ A(t) = 4(-1)^j (1 + zt) e^{-ixt} \left( \frac{W_1(t-y)}{W_2(t-y)} \right)^j \frac{E^{j-M}_2(t, q_m) E^{M-1}_2(t, q_n)}{E^{j-M+2}_1(t, q_n)} , \]  

(40)

and

\[ B(t) = (E_1(t, q_m))^{M-1} . \]

Taking high order derivatives of products of several functions (such as \( A(t) \)) analytically is very tedious, but such derivatives can easily be evaluated numerically on a computer by repeated application of Liebnitz' s rule:

\[ (gh)^{(k)} = \sum_{m=0}^{k} \frac{k!}{m! (k-m)!} g^{(m)} h^{(k-m)} , \]  

(41)

where \( g \) and \( h \) are functions of \( t \) and

\[ g^{(m)} = \frac{d^m}{dt^m} g . \]
In order to apply (41) to (39) and (40), we must be able to compute all derivatives of \((E_1(t, q))^{-1}\). In (41), let \(h = g^{-1}\) so that \(gh = 1\), and \((gh)^{(k)} = 0,\) if \(k > 0\). Then (41) becomes

\[
0 = g^{(0)} h^{(k)} + \sum_{m=1}^{k} \frac{k!}{m! (k-m)!} g^{(m)} h^{(k-m)} .
\]

Notice that \(g^{(0)} = g = 1/h\). Solve for \(h^{(k)}\):

\[
h^{(k)} = (g^{-1})^{(k)} = -h \sum_{m=1}^{k} \frac{k!}{m! (k-m)!} g^{(m)} h^{(k-m)} ,
\]

which is a recursive formula for derivatives of \(g^{-1}\) in terms of derivatives of \(g\).

Similarly, recursive formulas for the \(b_i\) for a pole of order \(N\) are:

\[
b_0 = (E_1^{(1)}(t_0, q_p))^{-1} ,
\]

where \(p = e\) or \(m\), as appropriate, and

\[
b_i = (E_1^{(1)}(t_0, q_p))^{-1} \sum_{k=1}^{i} \frac{k(N+1) - i}{i(k+1)!} b_{i-k} E_1^{(k+1)}(t_0, q_p) .
\]

Finally, all derivatives of the functions involved in (39) and (40) can be found recursively. Assume that \(W_k(t)\) and \(W_k'(t)\) are given. Then the differential equation for \(W_k\) \([W_k(t) = t W_k(t)]\) and straightforward mathematical induction show that

\[
W_k^{(n)}(t) = t W_k^{(n-2)} + (n-2) W_k^{(n-3)}(t) , \text{ for } n \geq 2 .
\]

Using (32),

\[
E_k^{(n)}(t, q) = W_k^{(n+1)}(t) - q W_k^{(n)}(t) .
\]
Of course, 
\[ (e^{-ixt})^n = (-ix)^n e^{-ixt}. \]  

(46)

We now return to the problem of finding the location of the poles, that is, the zeroes of \( E_1(t, q) \). We use Newton's iteration:

\[ t \approx \hat{t} - \frac{E_1'\hat{t}, q}{E_1(\hat{t}, q)} = \hat{t} - \frac{W_1'\hat{t}) - q W_1(\hat{t})}{\frac{i}{\hat{t}} W_1(\hat{t}) - q W_1(\hat{t})}, \]

where \( \hat{t} \) is an approximation to \( t \). For first approximations, we use the known zeroes of \( W_1'(t) \), if \( q \) is small, and the zeroes of \( W_1(t) \) if \( q \) is large. The first ten such zeroes are listed in Subroutine TW, which also contains approximate formulas for all the other zeroes.

3.4 Ionospheric Reflection Coefficients

The ionospheric reflection coefficients, \( T \), are functions of the angle of incidence, \( \varphi \), which depends on distance and the hop number. When the geometrical-optics formula (19) is used, \( \varphi \) is computed with (22). When (28) or (31) is used, \( \varphi \) is given by (29), \( \cos \tau_1 = \frac{-t_0^2}{v} \), and \( \sin \varphi = a_1/a_2 \sin \tau_1 \). When \( I_{jm} \) is evaluated with the residue series (33), we use these same formulas except now to is the first \( q_e \) pole, and hence is complex. Continuation of the ionospheric reflection coefficients into the complex \( \varphi \)-plane is accomplished by approximating the input values along the real axis with analytic functions and continuing these functions analytically.

Following Wait (1962), we approximate \( T_{ee} \) and \( T_{mm} \) by an exponential

\[ T \approx \exp(A \cos \varphi), \]

where \( A \) is a complex number determined from the input data:

\[ \text{Real} (A) = \log (|T|)/\cos \varphi, \]

(49)

\[ \text{Imag} (A) = (\text{Phase} (T) - \tau)/\cos \varphi. \]
The polarization coupling coefficients $T_{em}$ and $T_{me}$ are approximated by fitting their amplitude and phase with linear functions.

4. NOTES ON COMPUTER IMPLEMENTATION

4.1 Program Organization and Flow

Figure 2 shows the large-block program organization and flow. The numbers in the blocks are the statement numbers where the block function begins. Details of program organization and flow are best determined from the statements and comments in the program listing in section 4.4.

4.1.1 Numerical Integration Branch.

We use 48-point Gaussian quadrature to integrate (31) on a finite portion of the contour $\Gamma$. First, for $t > 1$,

$$W_k(t) \approx t^{-\frac{3}{8}} \exp\left(\frac{2}{3} \frac{3}{4} t^\frac{3}{8}\right),$$

and

$$W'_k(t) \approx t^\frac{1}{8} W_k(t),$$

so the magnitude of the integrand in (31) is about $4t^\frac{3}{8} \exp(-\frac{4}{3} t^\frac{3}{8})$. We consider the contribution beyond $t = 4$ to be negligible, so the first portion of the contour of integration runs from 4 to 0 along the real axis.

The second portion of the contour is a straight line from 0 to $K (-4 - i)$, where $K$ is chosen so that the real part of $-ixt$ is -9 for the shortest distance for which numerical integration would be used. Specifically, $K = -9/(2 \sqrt{4+y-4})$. The slope of this contour was chosen empirically to be optimum for convergence of the integrand. The rest of the infinite contour $\Gamma$ contributes little to the integral, so we ignore it.

4.1.2 Residue Series and the Subroutine LPAINR.

The residues are calculated in the subroutine LPAINR. Since only the factor $e^{-ixt}$ in $A(t)$ (see (39) and (40)) depends on distance, all necessary derivatives of $A(t)/e^{-ixt}$ are computed and stored on the
first entry to LPAINR. The $b_i$ \((43)\) are also independent of distance and are computed and stored on this first entry. Then, on every entry, $e^{-ixt}$ and its derivatives are computed and the derivatives of the product $e^{-ixt} (A(t)/e^{-ixt}) = A(t)$ are computed using \((41)\). The residues are then computed with \((38)\).

The storage in LPAINR is set up for at most 5 hops since the sixth hop would not normally use LPAINR except for paths longer than 10,000 km. Because quantities (such as the $b_i$) need to be stored only for $M \leq j$ (and for $M > 1$ for the $q_e$ poles), we save some storage by overlaying the arrays for the $q_e$ poles on those for the $q_e$ poles. If necessary, the details can be determined by examining the FORTRAN listing.

4.2 Input Data and a Test Case

Table 1 lists the input data necessary for each case, and figure 3 shows the arrangement of an input data deck. You can stack as many cases in sequence as you want. Execution terminates when an end-of-file card is read by statement 10.

Input data for a test case are listed in figure 4. The reflection coefficients used in this test case were computed with the program ANIREF, which is described in the appendix.

The printout produced by the test case is listed in figures 5-13. The "PHI C" printed out is the "phase lag" related to the phase of the field by

$$\varphi_c = -[\text{phase (E)} + kd + \pi/2],$$

where phase (E) is the phase of the total field or any of its components, as appropriate.

4.3 Important Compiler Characteristics

In our FORTRAN system, the variable index of a DO loop is compared with the maximum value before the DO loop is executed, and if
the index exceeds the maximum value, the loop is skipped. This cor-
responds with the usual mathematical convention that a sum or product
is empty if the lower limit of the index exceeds the upper limit, and
we have found it convenient to use this feature in the program and its
subroutines. If your compiler tests the index of the DO loop after
execution, you will need to modify the program accordingly.

Our computer stores two-dimensional arrays columnwise. That
is, if A(i,j) is a two-dimensional array, all of the elements of the column
j=1 are stored in sequence, followed by all the elements of the column
j=2, etc. A three-dimensional array can be thought of as an ordered
sequence of two-dimensional arrays, with the third subscript being the
number of the two-dimensional array. Therefore, a three-dimensional
array can be used in place of a two-dimensional array by fixing the value
of the third subscript. Similarly, a two-dimensional array can be
treated as a one-dimensional array by fixing the value of the second sub-
script. We have used this feature extensively in subroutine LPAINR.

4. 4 FORTRAN Listing of PROGRAM ANIHOP and Its Subroutines

See FORTRAN listing of ANIHOP beginning on page 33.

5. ACKNOWLEDGMENTS

Revision of the propagation program was sponsored by the Defense
Atomic Support Agency and monitored by Mr. Dow Evelyn. Mrs. Mary
Chrisman wrote the early versions of the program and several of its
subroutines. Dr. George Hufford originated the AIRY function sub-
routine. The reflection coefficient program in the appendix incorporates
many improvements developed by Dr. Bernard Wieder.

We would especially like to thank Mr. David Morfitt of NWC,
Corona; Dr. Gary Price of SRI; Mr. Burt Gambill of GE Tempo; and
Dr. G. Gonzales of University of Miami for many suggestions and dis-
cussions of the earlier version of the program.
Table 1. Input Data for the Program ANIHOP

<table>
<thead>
<tr>
<th>Variable</th>
<th>Physical Quantity</th>
<th>Units</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNDK</td>
<td>mho/m</td>
<td></td>
<td>Ground conductivity of path.</td>
</tr>
<tr>
<td>EPS2</td>
<td></td>
<td></td>
<td>Ground dielectric constant relative to free space.</td>
</tr>
<tr>
<td>DMIN</td>
<td>Distance</td>
<td>km</td>
<td>Minimum distance at which field is calculated.</td>
</tr>
<tr>
<td>DELTA</td>
<td>Distance</td>
<td>km</td>
<td>Distance increment.</td>
</tr>
<tr>
<td>DMAX</td>
<td>Distance</td>
<td>km</td>
<td>Largest distance for which NHOP hops will be used.</td>
</tr>
<tr>
<td>NHOP</td>
<td>Number of hops</td>
<td></td>
<td>See preceding comment.</td>
</tr>
<tr>
<td>NT</td>
<td>Number</td>
<td></td>
<td>Reflection coefficients are read in for this many angles of incidence.</td>
</tr>
<tr>
<td>POWER</td>
<td>Radiated power</td>
<td>watts</td>
<td>CCIR definition.</td>
</tr>
<tr>
<td>KASE</td>
<td></td>
<td></td>
<td>Controls punching of output cards; 1 means punch cards; 0 means no cards output.</td>
</tr>
<tr>
<td>ICOND</td>
<td>Number</td>
<td></td>
<td>Number of distance blocks to follow. May be 0.</td>
</tr>
<tr>
<td>XD(I)</td>
<td>Distance</td>
<td>km</td>
<td>NH(I) hops will be calculated for XD(I-1) to XD(I), where NH(0) = NHOP, and XD(0) = DMAX.</td>
</tr>
<tr>
<td>NH(I)</td>
<td>Number of hops</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ID(I), ITIM, IPHI, PHIA</td>
<td>Various (see comments)</td>
<td></td>
<td>Identification of ionospheric reflection coefficients; such as time, magnetic field, azimuth, etc.</td>
</tr>
<tr>
<td>FREQ</td>
<td>Radio frequency</td>
<td>kHz</td>
<td></td>
</tr>
</tbody>
</table>
### Table 1. Input Data for the Program ANIHOP (continued)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Physical Quantity</th>
<th>Units</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHI(N)</td>
<td></td>
<td>Degrees</td>
<td>Angle of incidence on ionosphere</td>
</tr>
<tr>
<td>TAMP(L, N)</td>
<td>T</td>
<td></td>
<td>Ionospheric reflection coefficient for PHI(N):</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L = 1$ for $T_{ee}$, $L = 2$ for $T_{em}$, $L = 3$ for $T_{me}$, $L = 4$ for $T_{mm}$</td>
</tr>
<tr>
<td>TPHA(L, N)</td>
<td>Phase of T</td>
<td>radians</td>
<td></td>
</tr>
<tr>
<td>HBOT</td>
<td>Height</td>
<td>km</td>
<td>The input phase(T) is referenced to this height.</td>
</tr>
<tr>
<td>HREF</td>
<td>Height</td>
<td>km</td>
<td>Effective height of reflection--phase(T) will re-referenced to this height in program.</td>
</tr>
</tbody>
</table>
Figure 3. Data deck set-up for program Anihop. The top line on the dummy card is a descriptive label; it should not be punched on the card.
Figure 4. List of input data deck for ANIHOP sample case.
**WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR 1000 WATTS RADIATED POWER**

**FREQUENCY = 30.00KHZ, EARTH CONDUCTIVITY = 4.0000 MHO/M, RELATIVE PERMITIVITY = 80.00**

**IONOSPHERIC MODEL**

**SAN FRAN SEAPATH NIGHT**

<table>
<thead>
<tr>
<th>PHI</th>
<th>COS(PHI)</th>
<th>AMPLITUDE</th>
<th>PHASE</th>
<th>AMPLITUDE</th>
<th>PHASE</th>
<th>AMPLITUDE</th>
<th>PHASE</th>
<th>AMPLITUDE</th>
<th>PHASE</th>
<th>AMPLITUDE</th>
<th>PHASE</th>
<th>HBOT</th>
<th>HREF</th>
</tr>
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<tbody>
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<td>65.00</td>
<td>0.4226</td>
<td>0.23860</td>
<td>3.860</td>
<td>0.18300</td>
<td>0.895</td>
<td>0.07000</td>
<td>-2.829</td>
<td>0.33320</td>
<td>0.864</td>
<td>74.0</td>
<td>76.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>73.00</td>
<td>0.2924</td>
<td>0.16270</td>
<td>4.149</td>
<td>0.20300</td>
<td>2.259</td>
<td>0.07230</td>
<td>-0.527</td>
<td>0.43830</td>
<td>1.889</td>
<td>74.0</td>
<td>74.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>81.00</td>
<td>0.1736</td>
<td>0.24100</td>
<td>3.163</td>
<td>0.20100</td>
<td>3.100</td>
<td>0.10270</td>
<td>0.427</td>
<td>0.67810</td>
<td>2.573</td>
<td>74.0</td>
<td>77.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>82.00</td>
<td>0.1392</td>
<td>0.28660</td>
<td>3.134</td>
<td>0.20050</td>
<td>3.178</td>
<td>0.10100</td>
<td>0.505</td>
<td>0.70730</td>
<td>2.644</td>
<td>74.0</td>
<td>77.6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**DISTANCE TOTAL GR WAVE HOP 1 HOP 2 HOP 3 HOP 4 HOP 5 HOP 6 HOP 7 HOP 8 HOP 9**

| 600 AMP | 2.75-004 | 3.44-004 | 1.50-004 | 8.31-005 |
| 800 AMP | 1.99-004 | 2.08-004 | 1.30-004 | 3.06-005 |
| 1000 AMP | 2.49-004 | 1.32-004 | 1.27-004 | 1.18-005 |
| 1200 AMP | 2.07-004 | 8.50-005 | 1.28-004 | 5.27-006 | 6.56-006 | 5.85-006 | 6.72-139 |
| 1400 AMP | 1.88-004 | 5.57-005 | 1.24-004 | 1.17-005 | 3.45-006 | 2.96-006 | 3.35-006 |
| 1600 AMP | 1.45-004 | 3.68-005 | 1.13-004 | 1.48-005 | 2.51-006 | 1.93-006 | 1.32-006 |
| 1800 AMP | 9.59-004 | 2.49-005 | 9.88-005 | 1.68-005 | 3.63-006 | 1.49-006 | 8.53-007 |
| 2000 AMP | 7.35-004 | 1.65-005 | 8.35-005 | 1.86-005 | 4.36-006 | 1.40-006 | 6.60-007 |
| 2200 AMP | 7.12-005 | 1.11-005 | 7.06-005 | 1.95-005 | 4.56-006 | 1.47-006 | 5.94-007 |
| 2400 AMP | 7.66-005 | 7.51-005 | 5.76-005 | 1.97-005 | 4.52-006 | 1.51-006 | 5.66-007 |
| 2600 AMP | 6.54-005 | 5.11-006 | 4.56-005 | 1.94-005 | 4.37-006 | 1.45-006 | 5.97-007 |

Figure 5. Printed output from ANIHOP sample case, page 1.
<table>
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<tr>
<th>AMP</th>
<th>PHI C</th>
<th>2800</th>
<th>3000</th>
<th>3200</th>
<th>3400</th>
<th>3600</th>
<th>3800</th>
<th>4000</th>
<th>4200</th>
<th>4400</th>
<th>4600</th>
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<th>5200</th>
<th>5400</th>
<th>5600</th>
<th>5800</th>
<th>6000</th>
<th>6200</th>
<th>6400</th>
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<tr>
<td>5.27-005</td>
<td>1.13</td>
<td>2.62</td>
<td>1.16</td>
<td>1.21</td>
<td>-1.52</td>
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Figure 6. Printed output from ANIHOP sample case, page 2.
Figure 7. Printed output from ANIHOP sample case, page 3.
WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR 1000 WATTS RADIATED POWER

FREQUENCY = 30.00KHZ, EARTH CONDUCTIVITY = 4.0000 MHO/M, RELATIVE PERMITIVITY = 80.00

IONOSPHERIC MODEL

SAN FRAN SEAPATH NIGHT PHI= 258.1 DIP= 39.0 HMM= 0.370 GAUSS REFERENCE HEIGHT = 78.200

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Figure 8. Printed output from ANIHOP sample case, page 4.
| 3000 AMP | 2.40E-005 | 2.38E-006 | 1.50E-005 | 4.24E-005 | 1.40E-005 | 5.19E-006 | 3.41E-006 |
| PHI C | 0.02 | 2.62 | 1.45 | -0.75 | 1.45 | -2.77 | 2.92 |
| 3200 AMP | 4.04E-005 | 1.63E-006 | 1.15E-005 | 3.01E-005 | 1.32E-005 | 5.03E-006 | 2.93E-006 |
| PHI C | -0.11 | 3.03 | 1.63 | -0.77 | 0.50 | 1.41 | -0.45 |
| 3400 AMP | 4.17E-005 | 1.12E-006 | 8.74E-006 | 3.50E-005 | 1.28E-005 | 4.96E-006 | 2.64E-006 |
| PHI C | -0.44 | -3.05 | 1.60 | -0.73 | -0.12 | -0.38 | 2.90 |
| 3600 AMP | 3.81E-005 | 7.70E-007 | 6.59E-006 | 3.08E-005 | 1.27E-005 | 4.80E-006 | 2.50E-006 |
| PHI C | -0.64 | -2.85 | 1.99 | -0.64 | -0.68 | -1.87 | 1.23 |
| 3800 AMP | 3.18E-005 | 5.30E-007 | 4.95E-006 | 2.65E-005 | 1.28E-005 | 4.43E-006 | 2.39E-006 |
| PHI C | -0.80 | -2.64 | 2.17 | -0.91 | -1.11 | -3.11 | -1.12 |
| 4000 AMP | 2.74E-005 | 3.66E-007 | 3.69E-006 | 2.23E-005 | 1.32E-005 | 4.18E-006 | 2.32E-006 |
| PHI C | -0.76 | -2.44 | 2.36 | -0.36 | -1.43 | 2.09 | 3.11 |
| 4200 AMP | 1.96E-005 | 2.53E-007 | 2.75E-006 | 1.80E-005 | 1.39E-005 | 4.04E-006 | 2.28E-006 |
| PHI C | -0.57 | -2.23 | 2.55 | -0.22 | -1.63 | 1.15 | 1.32 |
| 4400 AMP | 2.12E-005 | 1.75E-007 | 2.04E-006 | 1.42E-005 | 1.46E-005 | 3.97E-006 | 2.26E-006 |
| PHI C | -0.73 | -2.03 | 2.75 | -0.08 | -1.73 | 0.34 | -0.26 |
| 4600 AMP | 2.02E-005 | 1.21E-007 | 1.50E-006 | 1.10E-005 | 1.51E-005 | 4.01E-006 | 2.05E-006 |
| PHI C | -1.03 | -1.82 | 2.94 | 0.07 | -1.75 | -0.36 | -1.60 |
| 4800 AMP | 1.77E-005 | 8.42E-008 | 1.11E-006 | 8.59E-006 | 1.51E-005 | 4.15E-006 | 1.49E-006 |
| PHI C | -1.23 | -1.62 | 3.14 | 0.26 | -1.70 | -0.95 | -2.80 |
| 5000 AMP | 1.53E-005 | 5.84E-008 | 8.14E-007 | 6.67E-006 | 1.46E-005 | 4.30E-006 | 1.77E-006 |
| PHI C | -1.31 | -1.41 | 2.95 | 0.45 | -1.61 | -1.43 | 2.42 |
| 5200 AMP | 1.38E-005 | 4.06E-008 | 5.97E-007 | 5.14E-006 | 1.38E-005 | 4.45E-006 | 1.69E-006 |
| PHI C | -1.30 | -1.21 | 2.75 | 0.70 | -1.48 | -1.83 | 1.47 |
| 5400 AMP | 1.29E-005 | 2.63E-008 | 4.37E-007 | 3.94E-006 | 1.26E-005 | 4.73E-006 | 1.64E-006 |
| PHI C | -1.27 | -1.00 | 2.55 | 0.92 | -1.32 | -2.17 | 0.62 |
| 5600 AMP | 1.19E-005 | 1.97E-008 | 3.20E-007 | 3.00E-006 | 1.13E-005 | 5.22E-006 | 1.63E-006 |
| PHI C | -1.29 | -0.80 | 2.36 | 1.14 | -1.15 | -2.44 | -0.12 |
| 5800 AMP | 1.05E-005 | 1.37E-008 | 2.34E-007 | 2.28E-006 | 9.02E-006 | 5.91E-006 | 1.63E-006 |
| PHI C | -1.37 | -0.99 | 2.16 | 1.36 | -0.97 | -2.60 | -0.78 |
| 6000 AMP | 9.02E-006 | 9.57E-009 | 1.70E-007 | 1.72E-006 | 8.54E-006 | 6.63E-006 | 1.65E-006 |
| PHI C | -1.52 | -0.39 | -1.96 | 1.58 | -3.77 | -2.60 | -1.37 |
| PHI C | -1.77 | -0.18 | -1.76 | 1.80 | -0.56 | -2.67 | -1.06 |
| 6400 AMP | 6.23E-006 | 4.67E-009 | 9.03E-008 | 9.74E-007 | 5.95E-006 | 7.62E-006 | 1.81E-006 |
| PHI C | -1.88 | 0.02 | -1.56 | 2.02 | -0.35 | -2.60 | -2.30 |

Figure 9. Printed output from ANIHOP sample case, page 8.
Figure 10. Printed output from ANIHOP sample case, page 9.
**WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR 1000 WATTS RADIATED POWER**

**FREQUENCY = 30.00 KHZ, EARTH CONDUCTIVITY = 4.0000 MHO/M, RELATIVE PERMITIVITY = 80.00**

**IONOSPHERIC MODEL**

**SAN FRAN SEAPATH**

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Figure 11. Printed output from ANIHOP sample case, page 7.
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Figure 12. Printed output from ANIHOP sample case, page 5.
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Figure 13. Printed output from ANIHOP sample case, page 6.
6. REFERENCES


Davies, K. (1965), Ionospheric Radio Propagation, NBS Monograph 80.


33


PROGRAM ANIHOP

PROGRAM TO COMPUTE LF/VLF RADIO PROPAGATION USING WAVE HOP THEORY.


DIMENSION AMP(9), ATI(12), ATR(12), CPHI(13), E(9), FTX(2), G(48)
1+HOP(9,9), HREF(12), ID(10), JGO(9), NH(4), O(96), PH(9), PHI(12)
2+PT(45,96), Q(2), R(2,5,5), SD(96)+T(40), TA(9,4)
3+TAMP(4,13), TPHA(4,13), TPR(4,13), TPI(4,13), W(48), XD(4)

COMMON/PAIN/X,Y,Z,DMIN*XDIST*OSAV, A1, A2, Q, KMAX, T
COMMON/WGW/QKA1, QKA2, FK, AK1, V, V2, THETA, STH

COMPLEX B, C, C1, C2, DT1, DT2, ETA, F1, F2, F3, GW, HOP, O, PT, Q, KQA2, R, RE,
1+RM, RR, R1, R2, R2, R2, R2, R2, R3, R4, R5, S, SD, SQ2, SUM, S1, S2, T, TA, T1, T2, TY1, TY2

1 FORMAT(F5.1, F4.1, 2F6.1, F7.1, I2, I3, F7.1, I2, I2)
2 FORMAT(A8, A3, A8, 4X, A5, A6, A4, A8, * REFERENCE HEIGHT =*, F8.3)
3 FORMAT(F6.1, F4.1, 4(E9.3, F6.3), 2F5.1)
4 FORMAT(A8, A3, A8, 4X, A5, A6, A4, A8)
5 FORMAT(1H1, 10X, * WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD S
1+LENGTH (V/M) FOR *F8.0, * POWER*#/I0X,
2+HFREQUENCY = *F9.2KHZ, * EARTH CONDUCTIVITY =*, F8.4, * MHO/M,
3+RELATIVE PERMITIVITY =*, F7.0, *IONOSPHERIC MODEL/*
7 FORMAT(1H1, 8X, 5HWI49J, 15X, 5HW(50)/12E20.8)
8 FORMAT(1H1, 8X, 5HW(5)/12E20.8)
9 FORMAT(9X, *AMP *, 11E10.2)

ABSCISSA (G) AND WEIGHTS (W) FOR GAUSSIAN INTEGRATION

G(1) = -9987710073 $ G(2) = -9935301723 $ G(3) = -9841245837
G(4) = -9705915925 $ G(5) = -9529877032 $ G(6) = -9313866907
G(7) = -9058791367 $ G(8) = -8765720203 $ G(9) = -8435882616
G(10) = -8070662040 $ G(11) = -7671590325 $ G(12) = -7240341309
G(13) = -6778723796 $ G(14) = -6288673968 $ G(15) = -5772247261
G(16) = -5231609747 $ G(17) = -4669029048 $ G(18) = -4086846280
G(19) = -348755863 $ G(20) = -2837624874 $ G(21) = -2247637904
G(22) = -16122323561 $ G(23) = -9700469921 $ G(24) = -30038017096
G(25) = 03283017096 $ G(26) = 09700469921 $ G(27) = 16122323561
G(28) = 2247637904 $ G(29) = 2837624874 $ G(30) = 348755863
G(31) = 4086846280 $ G(32) = 4669029048 $ G(33) = 5231609747
G(34) = 5772247261 $ G(35) = 6288673968 $ G(36) = 6778723796
G(37) = 7240341309 $ G(38) = 7671590325 $ G(39) = 8070662040
G(40) = 8435882616 $ G(41) = 8765720203 $ G(42) = 9058791367
G(43) = 9313866907 $ G(44) = 9529877032 $ G(45) = 9705915925
G(46) = 9841245837 $ G(47) = 9935301723 $ G(48) = 9987710073
INPUT DATA

10 READ 1, CNDK, EPS2, DMIN, DELTA, DMAX, NHOP, NT, POWER, KASE, ICOND,
1(XD(I), NH(I))=1=I; ICOND)

C
C CNDK = GROUND CONDUCTIVITY IN MHOS/METER
C EPS2 = DIELECTRIC CONSTANT OF THE GROUND RELATIVE TO FREE SPACE
C NOTE. THE GROUND WAVE AND THE FIRST NHOP HOPS WILL BE
C CALCULATED EVERY DELTA KM FROM DMIN KM TO DMAX KM USING NHOP HOPS,
C THEN ON TO XD(1) KM USING NH(1) HOPS,
C THEN ON TO XD(2) KM USING NH(2) HOPS,
C AND SO ON TO ICOND SECTIONS. THE MAX OF NHOP OR NH IS 9.
C NT = NUMBER OF ANGLES OF INCIDENCE FOR WHICH THE REFLECTION
C COEFFICIENTS ARE READ INTO THE PROGRAM
C POWER IS THE RADIATED POWER IN WATTS RELATIVE TO DIPOLE IN HALF SPACE
C KASE = 1, THE AMPLITUDE AND PHASE OF THE WAVE HOPS ARE
C PUNCHED ON DATA CARDS - KASE = 0, NO DATA CARDS ARE OBTAINED

DS=-1.  
IF = 1

C CONTROL VARIABLES. DS IS DISTANCE MEMORY IN INTEGRATION LOOP.
C IF IS USED TO INDICATE FIRST OR SUBSEQUENT ENTRY TO CWGW0
C DSAV AND KMAX ARE USED IN LPAINR TO PREVENT RECALCULATION OF
C AVAILABLE NUMBERS.

DSAV=-1. $ KMAX=0
ICD=0
MHOP=NHOP
XDIST=DMAX
IF(EOF,60)999,20
20 IF(ICOND LE 0) GO TO 25
MHOP=NH(ICOND)
XDIST=XD(ICOND)

C IDENTIFICATION OF THE PROFILE
25 READ 4,1D(1),ID(2),ID(3),ITIM,IPH1,PHIA,ID(4),ID(5),ID(6),ID(7),
     7ID(8)
C THESE PARAMETERS ARE FOR IDENTIFICATION ONLY AND USUALLY
C INCLUDE A NAME, TIME, GEOGRAPHIC AZIMUTH, DIP ANGLE,
C AND MAGNETIC FIELD INTENSITY.
C READ ANISOTROPIC REFLECTION COEFFICIENTS

DO 30 N=1,NT
READ 3,FREQ,PHI(N),(TAMP(L,N),TPHA(L,N),L=1,4)*HBOT,HREF(N)

C FREQ = FREQUENCY IN KHZ
C PHI = ANGLE OF INCIDENCE IN DEGREES
C TAMP = AMPLITUDE OF REFLECTION COEFFICIENT
C TPHA = PHASE OF REFLECTION COEFFICIENT
C NOTE: THE REFLECTION COEFFICIENTS SHOULD BE GIVEN IN THE
C FOLLOWING ORDER- TEE, TEM, TME AND TMM
C HBOT = BOTTOM OF THE PROFILE IN KM
C HREF = REFLECTION HEIGHT IN KM

30 CPHI(N)=COSF(PHI(N).01745329252)
OEGA=FREQ.6283.185307
OKA1=WAVE*OEGA/2.997925E8
NPI=(CPHI(NT-1)*TPHA(NT-1)-CPHI(NT)*TPHA(NT-1))/
1(2.63185307*(CPHI(NT-1)-CPHI(NT))
HP=HREF(N)+NPI.2.63185307/(2.*WAVE*CPHI(N))*.001

C CALCULATION OF THE VARIABLES THAT ARE NOT A FUNCTION
C OF DISTANCE

AK1=AI*OKA1
A2=AI+H*1.E3
SQ2=CMLX(WAVE*WAVE*EPS2,-12.5663706E-7*OEGA*CNDK)
OKA2=CQRS(TSQ2)
R12=OKA1/OKA2
R21=OKA2/OKA1
V=CUBERTF(AK1/2.)
V2=V*V
Y=OKA1*H/1000./V

33 FK= 30.5 SQRT(3.141592653*POWER/120.)
Z=1.25/V2
Q(1)=V*R12*(0.,-1.)*CSQRT(1.-R12*R12)
ETA=R21*R21
Q(2)=ETA*Q(1)

C INITIALIZE
C NN=1 SIGNALS FIRST TIME IN NUMERICAL INTEGRATION

NN=1
DO 35 J=1,MHOP
AMP(J)=0.
35 PHI(J)=0.
C PRINT LABELS AND VALUES OF CONSTANTS

PRINT 5,POWER,FREQ,CNDK,EPS2
PRINT 2,ID(1),ID(2),ID(3),ITIM,IPHI,PHIA,ID(4),ID(5),ID(6),ID(7),
7ID(8),H
PRINT 36
36 FORMAT (*0 PHI 7* COS(PHI)*$, 7X*, T E E*, 15X*, T E M*, 15X*, T M
1E*, 15X*, T M M*, 12X*, HBOT*, 5X*, HREF/$
215X*, 4(* AMPLITUDE PHASE *))
C
C ADJUST PHASE OF T TO REFERENCE HEIGHT
DO 55 L=1,4
DO 40 N=1,NT
TPHA(L,N)=TPHA(L,N)+2*WAVE*(H-HB0T)*CPHI(N)*1000,
IF(TPHA(L,N)*GT. 6.283185307) TPHA(L,N)=MODF(TPHA(L,N),
1 6.283185307)
40 CONTINUE
C
C MAKE PHASE OF REFLECTION COEFFICIENT CONTINUOUS AS A FUNCTION
C OF COS(PHI)
CALL GUDFAZIL,CPHI,TPHA,NT)
C
C COMPUTE A FOR THIS ANGLE
C
DO 50 N=1,NT
IF ( L .EQ. 4 ) GO TO 47
IF ( L .EQ. 2 OR L .EQ. 3 ) GO TO 47
IF ( L .EQ. 4 ) PRINT 37, PHI(N), CPHI(N), (TAMP(I,N),TPHA(I,N),I=1,4),HBOT,HREF(N)
37 FORMAT ( F7.2, F8.4, 4(F10.5, F10.3), 2F10.1)
45 TPI(L,N)=LOGF(TAMP(L,N))/CPHI(N)
47 TPR(L,N)=TPHA(L,N)-3.141592653)/CPHI(N)
GO TO 50
47 TPR(L,N) = TAMP(L,N)
30 CONTINUE
50 CONTINUE
C
C PRINT 39
39 FORMAT (*ODISTANCE*,8X*,TOTAL*, 5X*, GR WAVE*, 3X*, HOP 1*, 5X,
1*HOP 2*, 5X*, HOP 3*, 5X*, HOP 4*, 5X*, HOP 5*, 5X*, HOP 6*, 5X,
2*HOP 7*, 5X*, HOP 8*, 5X*, HOP 9*)
C
C INITIALIZE JGO, METHOD SELECTOR FLAG FOR CALCULATING I SUB J,M
C
C GEOMETRICAL OPTICS FOR SHORTEST DISTANCES JGO=4
C SADDLE POINT FOR SHORT DISTANCES JGO=3
C NUMERICAL INTEGRATION FOR LONG DISTANCES JGO=2
C RESIDUE SERIES FOR LONGEST DISTANCES JGO=1
C
C (UNLESS N Hopkins GT. 5, THEN USE NUM. INTEG*)
C
DO 79 J=1,MHOP
IF(COSF(DMIN/(/0.02*J*A1)) *.LE. (A1/A2)) JGO(J)=1
CONTINUE

BEGINNING OF THE LOOP DMIN (DELTA) DMAX - LOOP ENDS AT 995

80 THETA=DMIN*1.E3/A1
X=V*THETA
X2=X*X
STH=SIN(THETA)
AK1D=QKA1*DMIN*1000.*
F3=FK/SQRT(A1*A1*THETA*STH)
F2= SQRT(2.*X/3.*141592653)*F3*(-1.*1.)
F1= (0.*6.*283185907) *F2
DO 85 J=1,NHOP
DO 85 M=1,J
85 HOP(J,M)=0

CALCULATION OF E SUB Q, THE GROUND WAVE

CALL CWGW( IF , DMIN, 0.,0.,1., GW , Q(1), X,Z)
SUM = GW

BEGIN HOP LOOP

DO 980 J=1,NHOP
TUJ=2.*J
TOTJ=THETA/TUJ
GO TO 1700,500,300,100),JGO(J)

BEGIN GEOMETRICAL OPTICS METHOD OF CALCULATING I SUB J,M

DJ=TUJ*SQRTF(2.*A1*A2*QMCOS(TOTJ)+H*H*1.E6)
SINTAU=TUJ*A2*SIN(TOTJ)/DJ
COSSQ=1.-SINTAU*SINTAU
COSTAU=SQRT(COSSQ)
COSPHI=(DJ+TUJ*A1*COSTAU)/(TUJ*A2)
COS5=COSPHI*COSTAU

IS COS TAU SMALL ENOUGH TO USE SADDLEPOINT FORMULA

IF(COS5 *.LE. 1./AK1) GO TO 295

150 BJ=A2/A1*SQRTF(DJ*SINTAU*COSPHI/(A2*STH*COSTAU))
S=(0.*1.)*V*COSTAU
C1 = CMPLX(COSPHI,0.)
RM=(S+Q(2))/(S-Q(2))
RE=(S+Q(1))/(S-Q(1))
P= QKA1*DJ-AK1D
SI=BJ*SINTAU*SIINTAU* FK/DJ*(1.*RE)**2
1 * CMPLX(COS(P),-SIN(P))
HOP(J,1) = SI*RE**(J-1)
DO 175 M=2,J

39
175 \texttt{HOP(J,M) = HOP(J,M-1)*RM/RE} \\
\texttt{GO TO 900}

\texttt{C 295 JGO(J) = JGO(J) - 1}

\texttt{C C BEGIN SADDLE POINT METHOD OF CALCULATING I SUB J,M}

\texttt{C 300 P = (4.*J*J*Y-X2)/(4.*X*J)}

\texttt{C IS P SO SMALL WE MUST INTEGRATE NUMERICALLY}

\texttt{IF (P \texttt{**2} + (J-1.)**15) GO TO 495}

\texttt{301 P2 = P*P}

\texttt{DJ = SQRT(Y+P2)}

\texttt{C1 = CMPLX(DJ/V,0.0)}

\texttt{OMEGA = -X*P2 + 1.333333*J*((Y+P2)*DJ-P2*P)}

\texttt{S2 = CMPLX(0.,P)}

\texttt{RE = (S2+Q(1)) / (S2-Q(1))}

\texttt{RM = (S2+Q(2)) / (S2-Q(2))}

\texttt{S = SQRT((1.+X/(2.*J*P))*(1.+5.*P2/V2)**5)*(1.+RE)**2}

\texttt{1.*CMPLX(COS(OMEGA), -SIN(OMEGA))*RE**J-1)}

\texttt{HOP(J,1) = F3*S}

\texttt{DO 475 M = 2*J}

\texttt{475 HOP(J,M) = HOP(J,M-1)*RM/RE}

\texttt{GO TO 900}

\texttt{C 495 JGO(J) = JGO(J) - 1}

\texttt{C C THE PATH INTEGRALS ARE CALCULATED USING NUMERICAL}

\texttt{C INTEGRATION. GAUSSIAN INTEGRATION}

\texttt{C IS USED WITH 48 ABSCISSAS AND WEIGHTS. THE CONTOURS}

\texttt{C OF INTEGRATION CONSIST OF TWO SEGMENTS, T GOING}

\texttt{C FROM 4 TO 0 AND THEN INTO THE LOWER HALF PLANE AT A}

\texttt{C SLOPE OF 1/4}

\texttt{C 500 IF (COSF(THEA/TUJ)) LE A1/A2) GO TO 695}

\texttt{C C IF WE ARE IN THE SHADOW REGION (TAU GE 90 DEGREES) USE RESIDUE SERIES.}

\texttt{501 IF (NN*EQ.2) GO TO 600}

\texttt{C C THE FIRST TIME A HOP IS CALCULATED WITH NUMERICAL}

\texttt{C INTEGRATION THE COEFFICIENT OF E**(-IXT) IS}

\texttt{C DETERMINED FOR THE GAUSSIAN ABSCISSAS AND STORED FOR}

\texttt{C FUTURE USE}

\texttt{T0P=0.0}

\texttt{BOT=4.0}

\texttt{KK=0}

\texttt{DO 590 L=1,2}

\texttt{DO 575 K=1,48}

\texttt{KK=KK+1}
IF(L·EQ·2) GO TO 520
OI=0.
OR=(TOP-BOT)*G(K)+TOP+BOT)*.5
GO TO 525
520 OI=((TOP-BOT)*G(K)+TOP+BOT)*.5
OR=O1*4.
525 O(KK)=CMPLX(OR, OI)
CALL CWairy(1. O(KK), T1, M1, T2, M2)
CALL CWairy(2. O(KK), DT1, MD1, DT2, MD2)
CALL CWairy(1. O(KK)-Y, TY1, MTY1, TY2, MTY2)
S2=DT1*(2*718281828**MD1)-G(1)*T1*(2*718281828**M1)
R1=(1+z*O(KK))/(S2*S2)
R2=2*718281828**(MTY1-MTY2)*TY1/TY2
R3=2*718281828**(MD2-M2)*DT2/T2
S1=2*718281828**(MD1-M1)*DT1/T1
R4=(S1-Q(1))/(S2-Q(1))
R5=(S1-Q(2))/(S2-Q(2))
S1=(1.1*O1)
RR=R1/R3
JM=0
DO 575 JJ=1, MHOP
RR=-RR*R2*R3
JM=JM+1
PT(JM, KK)=RR*S1
PT(JM, KK) = THE ARRAY OF COEFFICIENTS OF E**(-IXT)
FOR THE 48 GAUSSIAN ABSCISSAS
C NOW CALCULATE PT FOR ALL HOPS
C
S1= S1*R4
DO 574 M=2, JJ
MJ=JM
JM=JM+1
574 PT(JM, KK)=PT(MJ, KK)*R5/R4
575 CONTINUE
FTX(L)=.5*(TOP-BOT)
BOT=0.
590 TOP=-9.1/(2.*SQR(T(4.1+Y)-4.)
NN=2
600 KK=0
C CALCULATION OF THE INTEGRAL
DO 650 L=1, 2
DO 650 K=1, 48
HH=W(K)*FTX(L)
KK=KK+1
IF(DS .EQ. DMIN) GO TO 625
IF(L.EQ.2) GO TO 620
ARG=X*REAL(O(KK))
SD(KK)=CMPLX(COSF(ARG), -SINF(ARG))
GO TO 625
41
620 SD(KK)=EXP(COMPLEX(0.0,-X)*O(KK))
625 CONTINUE
   JM=(J*(J-1))/2
   DO 650 M=1,J
   JM=JM+1
   IF(L.EQ.2)GO TO 630
   SUMMING OF THE INTEGRAND FOR THE FIRST INTERVAL
   REAL PLANE
   HOP(J,M)=HOP(J,M)+HH*PT(JM,KK)*SD(KK)*F2
   GO TO 650
   SUMMING OF THE INTEGRAND FOR THE SECOND INTERVAL
   COMPLEX PLANE
   630 B=PT(JM,KK)*SD(KK)
   S1=B*(4.*1.*)
   HOP(J,M) = HOP(J,M) +HH*S1*F2
   CONTINUE
   ALP=(4.*J*Y-X2)/(4.*X*J)
   C1= COMPLEX(SQRT(Y+ALP*ALP)/V,0.0)
   DS=DMIN
   GO TO 900
   695 IF(J.GT.5) GO TO 501
   JGO(J)=JGO(J)-1
   THE PATH INTEGRALS ARE CALCULATED WITH THE RESIDUE
   SERIES
   700 MO=0
   800 DO 850 K=1,20
   LPAINR COMPUTES THE RESIDUES FOR ALL M FOR THIS J.
   CALL LPAINR(J,K,R)
   DO 825 M=1,J
   HOP(J,M) = HOP(J,M)-(R(1,J,M)+R(2,J,M))*F1
   IF(K.EQ.1)GO TO 825
   IF(CABS(((R(1,J,M)+R(2,J,M))*F1)/HOP(J,M)) .GT. 0.0005) GO TO 825
   MO=MO+1
   IF(MO.NE. J) GO TO 825
   GO TO 885
   825 CONTINUE
   850 CONTINUE
   885 CONTINUE
   DCRIT = SQRT(.008*A1*H)
   S1= MINIF(1.0, (DMIN-J*DCRIT)/(300*J))*T1
   COMPUTE TEE,TEM,TME AND TMM

42
900 CONTINUE
IF(JGO(J) .EQ. 1) GO TO 904
895 C1 = CMPLX((A1*OMCOS(TOTJ)+H*1000)/(2*A1*A2*OMCOS(TOTJ) + 1 H*H*1.E6),0)

C IF PHI IS MUCH SMALLER THAN ANY INPUT VALUE, SET TA (AND HENCE C THIS HOP) TO A VERY SMALL VALUE AS AN INDICATOR.

904 CONTINUE
IF(REAL(C1) .LT. 1.2*CPHI(1)) GO TO 905
TA(J,1)=TA(J,2)=TA(J,3)=TA(J,4)=1.E-25
GO TO 940
905 C2=C1*C1
NL=NT-1
NI=1 $ NIP = NI+1
CY = REAL(C1)

C DETERMINE WHICH VALUES OF T*COS(PHI) TO USE IN INTERPOLATION

906 IF((CY .LE. CPHI(NI)) .AND. (CY .GT. CPHI(NIP))) GO TO 908
IF(NI .GE. NL) GO TO 908
NI = NIP $ NIP = NIP +1
GO TO 906
908 CX = CPHI(NI) - CPHI(NIP)

C INTERPOLATE TO GET A SUB 1 AND A SUB 2

DO 925 L=1,4
ATR( 2) = ( TPR(L,NI) - TPR( L, NIP))/CX
ATI( 2) = ( TPI(L,NI) - TPI( L, NIP))/CX
ATR( 1) = TPR(L,NIP) -ATR( 2)*CPHI(NIP)
ATI( 1) = TPI(L,NIP) -ATI( 2)*CPHI(NIP)
IF(L.EQ.1.OR.L.EQ.4)GO TO 920
DY=ATI( 1)+ATI( 2)*REAL(C1)
DX=EXP(-ATI( 2)*AIMAG(C1))
TA(J,L) = (ATR( 1)+ATR( 2))*DX*CMPLX(COS(DY),SIN(DY))
GO TO 925
920 S=CMPLX(ATR( 1),ATI( 1))*C1+CMPLX(ATR( 2),ATI( 2))*C2
TA(J,L) = -CEXP(S)
925 CONTINUE
C CALCULATION OF C SUB J,M

940 E(J)=HOP(J+1)*TA(J,1)**J
DO 960 M=2,J
C=0.
AJMK=J-M+1
M1=M-1
DO 950 K=1,M1
IF(K.EQ.1)GO TO 950
AJMK=(J+2-M-K)*(M-K)/(K*(K-1))*AJMK
950 C =C +AJMK*TA(J,1)**(J+1-M-K)*(TA(J,2)*TA(J,3)**K*TA(J,4)
1**(M-1-K)
960 CONTINUE
CALCULATION OF E SUB J, THE WAVE HOPS.

960 E(J) = E(J) + HOP(J,M) * C

CALCULATION OF E SUB R, THE VERTICAL ELECTRIC FIELD

SUM = SUM + E(J)
AMP(J) = CABS(E(J))

980 PH(J) = -CANG(E(J))

END OF J LOOP

GAMP = CABS(GW)
GPH = -CANG(GW)
SAMP = CABS(SUM)
SPH = -CANG(SUM)

PRINT AND PUNCH OUTPUT

PRINT 8, DMIN, SAMP, GAMP, (AMP(J), J=1,NHOP)
PRINT 9, SPH, GPH, (PH(J), J=1,NHOP)

990 IF(KASE.EQ.0) GO TO 995
PUNCH 12*FREQ,ID(1),ID(2),ITIM,PHIA,DMIN,SAMP,CNDK
12 FORMAT(F7.1,2A8,2X,A8,A6,F9.2,E20.8,F6.1)

END OF LOOP DMIN (DELTA) DMAX

995 DMIN = DMIN + DELTA
IF(DMIN = DMAX) 80, 80, 1000
999 CALL EXIT
1000 IF(ICD .GE. ICOND) GO TO 10
ICD = ICD + 1
DMAX = XD(ICD)
NHOP = NH(ICD)
GO TO 80
END
SUBROUTINE LPAINR(J,K,R)
C
C SUBROUTINE FOR ANIHOP. COMPUTES RESIDUES FOR GIVEN J,K, AND
C DISTANCES.
C
C IMPORTANT
C ARRAYS E AND A MUST HAVE THE SAME FIRST DIMENSION FOR USE
C IN DEPROD
C
C TYPE INTEGER TUJ
C COMPLEX Q1,T1,W1,E1,A,B1M1,AJM1,SA1,DA1,EX1,AHAT1,R
DIMENSION Q(2),T(40),W(8,2,21),MW(20,2,4),TE(2),E(7,2,2),A(7,15)
1,BJ(6,20,6),AJM(7,20,15),DA(7,6),EX(6,40),AHAT(6),R(2,5,5)
COMMON/PAIN/X,Y,Z,DMin,XDIST,DSAV,A1,A2,Q(KMAX+T
COMMON/PROD/C
C COMPLEX C(7,15)
C DATA (EC=2.718281828l
C NP(N) = IZ**5+IS*N
C LP(L,N) = L+IZ*(IZ**5+IS*N)
C
C IF THIS IS THE FIRST ENTRY FOR THIS MODEL, COMPUTE JRES
I
IF(DSAV .GT. 0.) GO TO 5
JRES=MINOF(5*FIXXF(XDIST/(.002*A1*ACOSF(A1/A2))))
KEX=0
5 IO=1
C IQ=1 FOR Q SUB E POLES. IQ=2 FOR Q SUB M POLES.
C
C IZ=0
C IS=1
C NDER=JRES+1
C KK=K $ JN=J+1
C IW=1 $ IY=2
C SIGN=(-1)**(J-1)
C
C IF WE HAVE COMPUTED A S AND B S FOR THIS K, GO TO 240
C
10 IF(K .GT. KMAX) GO TO 240
C
C FROM HERE TO 200, COMPUTE AJM AND BJM FOR THIS K.
C
C FIRST, FIND T(K) AND AIRY FUNCTIONS OF T.
C
C TW FINDS SOLUTIONS OF E1(T,QQ)=0.
C
20 CALL TW(K-I,Q(IQ),T(KK),W(I1*I1),MW(K1*1,IW),W(2*I1),MD1,
C AW(1*2*1),MW(K2*2*1),W(2*2*1),MD2)
C TE(1)=T(KK)
C TE(2)=TE(1)**Y
C W(2*I1)=EC**(MD1-MW(K1*1,IW)))*W(2*I1)
C W(2*2*1)=EC**(MD2-MW(K2*2*1)))*W(2*2*1)
C CALL CWAIRY(1,TE(2),W(1*I2),MW(K1,1,IY),W(1*2*2),MW(K2,1,IY))
C CALL CWAIRY(2,TE(2),W(2*I2),MD1*W(2*2*2),MD2)
C W(2*I2)=EC**(MD1-MW(K1,1,IY)))*W(2*I2)
W(2,2,2) = (EC*(MD^2 - MW(K,2,Y))) * W(2,2,2)

NOW GET HIGHER DERIVATIVES OF W AND E

NTOP=NDER+2
DO 30 N=3,NTOP
DO 29 I=1,2
DO 28 KIND=1,2
W(N,KIND,I) = TE(I) * W(N-2,KIND,I) + (N-3) * W(N-3,KIND,I)
E(N-1,KIND,I) = W(N,KIND,I) - Q(I) * W(N-1,KIND,I)
IF(N GT 3) GO TO 28
E(I,KIND,I) = W(I,KIND,I) - Q(I) * W(I,KIND,I)
28 CONTINUE
29 CONTINUE
30 AIN=N-2,I=WIN-2,I

NOW SUB ONE (T-Y) AND DERIVATIVES ARE IN COL. 1 OF A.

NOW COMPUTE BJM

40 BJM(LP(1,1),K,NP(1)) = E(2,1,1)
DO 50 N=2,NDER
BJM(LP(1,N),K,NP(N)) = BJM(LP(1,N-1),K,NP(N-1)) * E(2,1,1)
DO 48 L=2,N
SUM=0.
KF=1
KKT=L-1
DO 46 KI=1,KKT
KF=KF*(KI+1)
SUM=(KI*(N+1)-KKT)/(KKT*KF) * BJM(LP(L,KI,N),K,NP(N)) *
1 E(KI+2,1,1)+SUM
46 CONTINUE
48 BJM(LP(L,N),K,NP(N)) = SUM/E(2,1,1)
50 CONTINUE

DERINV FINDS DERIVATIVES OF 1/F.

CALL DERINV(W(1,2,2),NDER-1,A(1,2))

GET W/W AND DERIVATIVES AND PUT IN COL. 2 OF A.

DEPROD COMPUTES DERIVATIVES OF A PRODUCT OF N FUNCTIONS.

CALL DEPROD(A,2,NDER-1,A(1,2))

NOW PUT D(2) AND DERIVATIVES IN COL. 1 OF A.

A(1,1)=1+2*TE(1)
A(2,1)=2
A(3,1)=A(4,1)=A(5,1)=A(6,1)=0.
GO TO (70,150),IQ
C    AJM FOR QE POLES

70 CONTINUE
   CALL DERINV(E(1,1,2),NDER-1,E(1,1,2))
   CALL DEPROD(E(1,1,2),2,NDER-1,A(1,15))
C     E2/E1 AND ITS DERIVATIVES ARE NOW IN COL 15 OF A.*
C     NOW CALCULATE AJM/EXP(-IXT)

JRES2 = JRES+JRES
DO 75 JX=4,JRES2+2
   DO 74 I=1,NDER
      A(I,JX) = E(I,2,1)
74   A(I,JX-1) = A(I,2)
75 CONTINUE
   DO 90 M=1,JRES
      CALL DEPROD (A*M,JRES2,JRES-M+1,DA)
      DO 80 JX=M,JRES
         JM= JX-5+(M*(11-M))/2
         IM= JX-M+2
      DO 78 I=1,IM
         AJM(I+JX,K,M) = C(I,JX+JX)
78    CONTINUE
   IF(M .EQ. JRES) GO TO 240
   JP = M+M+2
   IM=NDER-M
   DO 82 I=1,IM
      A(I,JP) = A(I,15)
82    CONTINUE
GO TO 240
C    AJM FOR QM POLE

150 AJM(3+K,6) = A(1+1)*E(1,2,2)*E(1,1,1)**2
   IF(JRES .LT. 3) GO TO 200
   ZX=A(1,2)*E(1,2,1)/E(1,1,1)
   CALL DERINV(E(1,1,1), NDER-1, E(1,1,1))
   DO 160 N=1,NDER
      A(N+4)=A(N+3)=A(N+2)
      A(N,6)=A(N,5)=E(N+1,1)
      A(N,8)=A(N,7)=E(N+2,2)
   JX=N+2
   IF(N .GE. NDER) GO TO 161
C     STATEMENT 160 GETS AJM SUB ZERO FOR J=N+1, M=2

160 AJM(1+JX, K, N+6) = AJM(JX,K,N+5) * ZX
161 CONTINUE
C    GET AJM SUB ZERO AND ONE FOR J=3, M=3
   CALL DEPROD(A*8,NDER-1,A(1,1))
   AJM(3,K+10)=A(1,1)

47
AJM(4, K, 10) = A(2, 1)  
IF(JRES .LT. 4) GO TO 200  
CALL DEPROD(E(1, 1, 1), 2, NDER-1, A(1, 5))  
DO 170 N = 1, NDER  
A(N, 3) = A(N, 5)  
170 A(N, 4) = A(N, 2)  
DO 190 M = 3, JRES  
JM = (M*(11-M))/2-1  
M = M-1  
175 CALL DEPROD(A(5, M-2, JM+1), 8-M, K, JM+1)  
IF(M .EQ. 5) GO TO 200  
DO 176 I = 1, M  
176 AJM(I+6-M, K, JM) = C(I, 3)  
178 MF = 2*M-3  
DO 180 N = 1, NDER  
180 A(N, MF) = E(N, 2, 2)  
190 CONTINUE  
200 CONTINUE  
KMAX = K  
240 IF(DSAV .NE. DMIN) GO TO 250  
IF(K .LE. KEX) GO TO 300  
C COMPUTE EXP(-IXT) AND DERIVATIVES.  
250 ARG = CMPLX(0., -X)  
EX(1, KK) = EXP(ARG*T(KK))  
DO 260 N = 2, NDER  
260 EX(N, KK) = ARG*EX(N-1, KK)  
IF(IQ .EQ. 2) KEX = K  
300 SCALE = 2.718281828**(J*(MW(K+1, IY) - MW(K, IY)) + (J-1)*(MW(K, IY)+A-MW(K, I+1, I)))  
A = MW(K, I, IW))  
C COMPUTE RESIDUES FOR I-SUB-JM FOR THIS POLE.  
DO 330 M = 1, J  
JB = J-M+2  
IN = 0  
IF(IQ .EQ. 1) GO TO 301  
IN = JB  
JB = M-1  
301 JM = J-5+(M*(11-M))/2  
IF(JB = 1) 302, 303, 304  
302 R(IQ * J + M) = 0.  
GO TO 330  
303 R(IQ, J + M) = SCALE*A JM(1+IN, K+ JM) * EX(1, KK) / BJM(LP(1, I) + K, NP(I))  
GO TO 330  
304 DO 305 N = 1, JB  
A(N, 1) = AJM(N+1, K, JM)  
305 A(N, 2) = EX(N, KK)  
C COMPUTES A-SUB-N, EQ(36).  
CALL DEPROD(A, 2, JB-1, DA)  
NPB = NP(JB)  
48
FACT=1.

C NOW COMPUTE A-HATS USING (38).

AHAT(1)=DA(1)/BJM(LP(1,JB),K,NPB)
DO 315 N=2,JB
DA(N)=DA(N)/FACT
SUM=DA(N)
IM=N-1
DO 312 L=1,IM
  SUM=SUM-BJM(LP(N-L+1,JB),K,NPB)*AHAT(L)
AHAT(N)=SUM/BJM(LP(1,JB),K,NPB)
315 FACT=FACT*N
R(IQ,J,M)=SCALE*AHAT(JB)
330 CONTINUE
IF(IQ .EQ. 2) GO TO 350
IQ=2
I2=1
IS=-1
KK=K+20
Iw=3
Iy=4
JN=J-1
NDER=JRES-1
GO TO 10
350 DSAV=DMIN
RETURN
END
SUBROUTINE DEPROD(A,JF,JN,DA)
C
INPUT
C A(I,J)=THE (I-1)TH DERIVATIVE (I=1,JN+1) OF FUNCTION A(I,J),(J=1,JF)
C
OUTPUT
C DA(I)=THE (I-1)TH DERIVATIVE OF THE PRODUCT A(1,1)*A(1,2)*...A(1,JF)
C
IMPORTANT
C INPUT AND OUTPUT ARRAYS IN DEPROD MUST HAVE SAME FIRST
C DIMENSION FOR USE WITH LPAINR
C
MAKES REPEATED USE OF LEIBNITZS RULE.

COMMON/PROD/C
COMPLEX A(7*15),DA(15),C(7*15)
N=JN+1
DO 25 N=1,N1
25 C(N,1)=A(N,1)
DO 50 M=2,JF
50 C(1,M)=C(1,M-1)*A(1,M)
DO 75 M=2,JF
75 C(N,M)=C(1,M-1)*A(N,M)
FACN=1
FACL=1
DO 73 L=2,N
FACN=FACN*(N-L+1)
FACL=FACL*(L-1)
73 C(N,M)=C(N,M)+FACN/FACL*C(L,M-1)*A(N-L+1,M)
C
C(N,M)=N TH DERIVATIVE OF PRODUCT (A(1,L),L=1,M)

74 CONTINUE
75 CONTINUE
DO 100 I=1,N1
100 DA(I)=C(I,JF)
RETURN
END
SUBROUTINE CWGW ( MM, DMIN, H1, H2, ALFA, GW, Q, X, Z)

C CALCULATION OF THE GROUND WAVE
C
C INPUT
C DMIN = DISTANCE BETWEEN TRANSMITTER AND RECEIVER IN KM
C H1 = HEIGHT OF THE TRANSMITTER IN KM
C H2 = HEIGHT OF THE RECEIVER IN KM
C ALFA = EFFECTIVE EARTH RADIUS
C Q = -I*V*(NORMALIZED SURFACE IMPEDANCE)
C X = V*THETA
C Z = 1.25/V**2

C OUTPUT
C GW = THE GROUND WAVE

COMMON/WGW/QKA1,QKA2,FK,AK1,V,V2, THETA,STH
COMPLEX (T200),W200),G,QKA2,R12,S,D,Q WL,DW1,LY1,LY2,GW*S2,TS
Y1=QKA1*H1*1000./V
Y2=QKA1*H2*1000./V
COTH=COS(THETA)/STH
FAC=FK*SQRT(6.283185307)*XI!16.36739E6*ALFA**2*THETA*STH)
GW=0.

C IF THIS IS THE FIRST ENTRY COMPUTE T-SUB-S AND
C COEFFICIENT OF EXP(-IXT)
GO TO(60,125),MM
60 J2=1
MM=2
65 DO 100 J=J2,200
C FIND LOCATION OF POLE.
CALL RW( J-1, Q, T(J), W1, MW1, DW1, MD1, S,M,S,M)
IF(H1.GT.0)GO TO 80
IF(H2.GT.0)GO TO 75
W(J)=1.
GO TO 85
75 CALL CWAIRY(1,T(J)-Y2*Wy2*MY2*S,M)
W(J)=2.7182818***(MY2-MW1)*WY2/W1
GO TO 85
80 CALL CWAIRY(1,T(J)-Y1*Wy1*MY1*S,M)
W(J)=2.7182818***(MY1-MW1)*WY1/W1
IF(H2.GT.0)GO TO 85
CALL CWAIRY(1,T(J)-Y2*Wy2*MY2*S,M)
S=2.7182818***(MY2-MW1)*WY2/W1
W(J)=W(J)*S
85 W(J)=W(J)*(1+Z*T(J))/(T(J)-Q*Q)
C COMPUTE TERM OF RESIDUE SERIES AND ADD.
G=(1.+(CMPLX(3.,COTH))/8.*(AK1+V*T(J)))*CEXP((0.,-1.)*X*T(J))
G=W(J)*G
GW=GW+G
IF(J.EQ.1)GO TO 100
IF(CABS(G/GW) GT 0.0005) GO TO 100
J1=J
GO TO 110
100 CONTINUE
J2=200
GO TO 165
110 IF(J1.LE.J2)GO TO 165
J2=J1
GO TO 165
125 DO 140 J=1,J2
G=(1.+(CMPLX(3.,COTH))/8.*(AK1+V*T(J)))*CEXP((0.,-1.)*X*T(J))
G=W(J)*G
GW=GW+G
IF(J.EQ.1)GO TO 140
IF(CABS(G/GW) LT 0.0005) GO TO 165
140 CONTINUE
IF(J2.GE.200)GO TO 165
J2=J2+1
GO TO 65
165 GW=GW*FAC *(1.,-1.)
RETURN
END

SUBROUTINE ZEXP(A,B,X,Y,MAGTUD)
C SCALED EVALUATION OF THE EXPONENTIAL FUNCTION IN THE COMPLEX PLANE.
C INPUT.
C (A+IB) = THE COMPLEX EXPONENT.
C OUTPUT.
C EXP(A+IB) = (X+IY)*(E**MAGTUD)
MAGTUD=A
SCALE=MAGTUD
E=EXP((A-SCALE)
X=E*COSF(B)
Y=E*SINF(B)
RETURN
END
FUNCTION OMCOS(X)

C OMCOS(X) = 1 - COS(X)

C IS ACCURATE FOR ALL X INCLUDING X NEAR 0.

IF(ABS(X) .GT. 15) GO TO 40
IF(X .EQ. 0.) GO TO 50

C IF X IS SMALL, SUM TAYLORS SERIES FOR 1-COS(X)

S = X*X
OMCOS = T = .5*S
R = 4.*

10 T = T*S/(R*(R-1.))
OMCOS = OMCOS + T
IF(ABS(T/OMCOS) .LE. 5E-9) GO TO 51
R=R+2.*
GO TO 10

40 OMCOS = 1.-COS(X)
RETURN

50 OMCOS = 0.*
51 RETURN

END

SUBROUTINE DERINV(F,N,EF)

C INPUT
C F(K) = THE (K-1)TH DERIVATIVE OF F, K=1,N+1

C OUTPUT
C EF(K) = THE (K-1)TH DERIVATIVE OF 1/F, K=1,N+1

COMPLEX F(15),DF(15),EF(15)
DF(1)=1./F(1)
DO 50 K=1,N
K1=K+1
FACK=1.*
FACM=1.*

DO 50 M=1,K
FACK=FACK*(K1-M)
FACM=FACM*M
50 DF(K1)=DF(K1)-(FACK/FACM*F(M+1)*DF(K1-M))/F(1)
M=N+1
DO 60 K=1,M
60 EF(K)=DF(K)
RETURN

END
SUBROUTINE Tw( I, Q, T, W1,MW1, DW1,MD1, W2,MW2, DW2,MD2)
C T IS THE I-TH ROOT OF W-SUB-ONE-PRIME - Q*W-SUB-1 =0. (W IS THE AIRY FUNCTION)
C THE ROOTS ARE COUNTED IN ORDER OF INCREASING MAGNITUDE.
C W-SUB-ONE(T) = EXP(MW1)*W1, W-SUB-ONE-PRIME = EXP(MD1)*DW1, ETC.
C DIMENSION TZERO(I), TINFIN(I)
COMPLEX Q*W1+DW1*W2+DW2, PH, A, T
C W-SUB-ONE-PRIME(TZERO(I)) =0.
DATA (TZERO= 1.018793, 3.2481975, 4.8200992, 6.1633074, 7.3721773, 8.4884868, 9.5354490, 10.52766, 11.475057, 12.384788, 13.262219)
C W-SUB-ONE(TINFIN(I)) =0.
DATA (PH = (0.5, -0.8660251)), ( CON = 1.17809724)
C IF REAL(Q)**2 + AIMAG(Q)**2 *GT. 1.) GO TO 50
IF(I *GT. 10) GO TO 10
TZ = TZERO(I+1)
GO TO 20
10 YS = ((4*I+1)*CON)**2
TZ = CUBERTF(YS)*((1.-1.458333/YS)
20 T = TZ*PH
C T IS NOW SOLUTION FOR Q =0. THE NEXT STEP IS THE FIRST NEWTON ITERATION.
T = T+Q/T
GO TO 100
50 IF(I *GT. 10) GO TO 60
TZ = TINFIN(I+1)
GO TO 70
60 YS = ((4*I+3)*CON)**2
TZ = CUBERTF(YS)*((1.+1.041667/YS)
70 T = TZ*PH
C T IS SOLUTION FOR Q=INFINITY. NEXT STEP IS THE FIRST NEWTON ITERATION.
T = T+1./Q
100 K=0
C NOW USE NEWTONS ITERATION TO CONVERGE ON SOLUTION
C CWAIRY COMPUTES W(T) AND W PRIME (T)
101 CALL CWAIRY(1*T,W1,MW1,W2,MW2)
CALL CWAIRY(2*T,DW1,MD1,DW2,MD2)
A=(2.718281828**(MD1-MW1))*DW1/W1
A = (A-Q)/( T -A*Q)
T = T - A
K = K + 1
IF(K * GT. 15) GO TO 150
IF(CABS(A/T) * GT. 0.5E-6) GO TO 101
RETURN
150 PRINT 155 * I, T, A
155 FORMAT('* FAILED TO CONVERGE ON T(*, I2, *), T = *,
1 \ C(E14.6,E14.6),* LAST CORRECTION =*, C(E14.6,E14.6))
RETURN
END

SUBROUTINE GUDFAZ(K, C, FAZ, N)
SMOOTHS PHASE OF IONO REFLECTION COEFFICIENTS FOR ANIHOP
K = 1, 2, 3, 4 FOR TEE, TEM, TME, TMM, RESPECTIVELY
C = COS(PHI), ARRANGED IN ORDER OF INCREASING PHI
(=ANGLE OF INCIDENCE)
FAZ = PHASE OF REFLECTION COEFFICIENT, ARRANGED IN
SAME ORDER AS C.
N = NUMBER OF DATA POINTS

DIMENSION C(20), FAZ(4, 20)
DATA (PI=3.141592653)
C(N+1) = 0.
FAZ(K*N+1) = PI
3 IF(ABS(PI-FAZ(K*N)) * LE. PI) GO TO 5
FAZ(K*N) = FAZ(K*N) - SIGN(2*PI, FAZ(K*N))
GO TO 3
5 DO 10 I = 2, N
J = N+2-I
TEST = PI + (FAZ(K, J+1) - FAZ(K, J)) * C(J-1) / (C(J+1) - C(J))
7 TRY = TEST - FAZ(K, J-1)
IF(ABS(TRY) * LE. PI) GO TO 8
FAZ(K, J-1) = FAZ(K, J-1) + SIGN(2*PI, TRY)
GO TO 7
8 TRY = FAZ(K, J-1) - FAZ(K, J)
IF(ABS(TRY) * LT. 2*PI) GO TO 10
FAZ(K, J-1) = FAZ(K, J-1) - SIGN(2*PI, TRY)
GO TO 8
10 CONTINUE
RETURN
END
SUBROUTINE CIAIRY(KK,T,F1,M1,F2,M2)

CALCULATION OF THE W(T) AIRY FUNCTIONS

INPUT

KK=1, W(T) OF KIND 1 AND W(T) OF KIND 2 ARE COMPUTED

KK=2, THE DERIVATIVE OF W(T) OF KIND 1 AND THE DERIVATIVE OF

W(T) OF KIND 2 ARE COMPUTED

T = THE COMPLEX ARGUMENT

OUTPUT

F1*(E**M1) = W(T) OF KIND 1 OR THE DERIVATIVE OF W(T) OF KIND 1

AS INDICATED BY KK

F2*(E**M2) = W(T) OF KIND 2 OR THE DERIVATIVE OF W(T) OF KIND 2

AS INDICATED BY KK

NOTE, F1 AND F2 ARE COMPLEX, E=2.718281828... AND M1 AND M2 ARE

EXPONENTS

COMMON/MEXP/M

COMPLEX F1,F2,W11,W12,W11P,W12P,T

GO TO(100,200),KK

100 F2=W11(T)

M2=M

F1=W12(T)

M1=M

GO TO 300

200 F2=W11P(T)

M2=M

F1=W12P(T)

M1=M

300 F1=1.7724538509*(0.,-1.)*F1

F2=1.7724538509*(0.,+1.)*F2

RETURN

END
COMPLEX FUNCTION AIRY(ZZ)

SUBROUTINE TO CALCULATE AIRY FUNCTIONS. USES TAYLORS EXPANSIONS
AROUND VARIOUS COMPLEX CONSTANTS FOR SMALL T AND ASSYMPTOTIC
SERIES FOR LARGE T. WRITTEN BY DR. GEORGE HUFFORD, ITS,
BOULDER, COLORADO 80302.

COMPLEX ZZ
COMMON/MEXP/M

COMPLEX Z1,A*AP*,U*Z1*,Z1*,ZB*,ZE*,ZB*,B0*,B1*,B2*,B3
DIMENSION XI(2),X1(2),XT(2) $ EQUIVALENCE (X,Z1),(X1,Z1),(XT,ZT)
DATA (LG=3)*(Z1=(0.,0.))*(A=(0.35502805389,0.))
DATA (AP=(-0.25881940379,0.))
DIMENSION AV(70) $ COMPLEX AV $ DATA ( AV=
X (-3.2914517363E-001, 0.0000000000E+000), AV 1
X (-2.6780035625E+000, 1.4774589547E+000), AV 2
X ( 3.5076100903E-001, 0.0000000000E+000), AV 3
X ( 2.4122262158E+000, 6.9865124448E-001), AV 4
X ( 3.3635531189E+001, 3.4600959696E+000), AV 5
X ( 3.4497396313E+002, 3.690892505E+002), AV 6
X ( 7.0265532950E-002, 0.0000000000E+000), AV 7
X ( 5.4818219290E+000, 9.207365909E+000), AV 8
X ( 3.383395342E+001, 6.022598082E+001), AV 9
X ( 2.2967795901E+002, 3.072452637E+002), AV 10
X ( 1.8040780476E+003, 2.197675036E+003), AV 11
X ( 1.7881429368E-001, 0.0000000000E+000), AV 12
X ( 1.3491836060E+000, 4.969077213E-001), AV 13
X ( 6.0453339320E+000, 1.062317554E+001), AV 14
X ( 3.1169621995E+001, 9.881351765E+000), AV 15
X ( 9.8925349347E+002, 1.390528608E+002), AV 16
X ( 2.2740472820E+001, 0.0000000000E+000), AV 17
X ( 7.1857403459E+000, 9.760909170E+000), AV 18
X ( 6.0621808063E+000, 2.720301486E+000), AV 19
X ( 3.6307084828E+000, 2.096135813E+000), AV 20
X ( 6.7139789190E+000, 3.040638708E+001), AV 21
X ( 2.8800165369E+000, 4.649363589E+000), AV 22
X ( 5.3556088329E-001, 0.0000000000E+000), AV 23
X ( 9.2407365385E-001, 1.910650605E-001), AV 24
X ( 1.871618961E+000, 2.574310394E-001), AV 25
X ( 7.2188435280E-001, 1.292420019E+000), AV 26
X ( 8.1787377840E+000, 3.087013839E+000), AV 27
X ( 2.9933948552E+000, 5.692217925E+000), AV 28
X ( 3.5502805389E-001, 0.0000000000E+000), AV 29
X ( 3.120348104E-001, 3.884538509E-001), AV 30
X ( 5.2839999360E-001, 1.097641122E+000), AV 31
X ( 4.2009351585E+000, 1.194015119E+000), AV 32
X ( 7.1858832892E+000, 1.960912513E+000), AV 33
X ( 1.0129121011E+000, 7.595123329E-002), AV 34
X ( 1.3529214631E-001, 0.0000000000E+000), AV 35
X ( 3.2618478398E-002, 1.708487278E-001), AV 36
X ( 3.4215381085E-001, 8.906764633E-002), AV 37
X ( 1.4509641493E-001, 1.032801574E+000), AV 38
X ( 4.1001968523E+000, 6.893691176E-001), AV 39
X 1.9293755496E+009, 2.1428803701E+008, 2.5198199876E+007,
X 3.1482574185E+006, 4.1952487519E+005, 5.9892513580E+004,
X 9.20720637E+003, 1.49352037E+003, 2.7846908084E+002,
X 5.62278537E+001, 1.234157335E+001, 3.0794530307E+000,
X 8.766696967E-001, 2.9159139927E-001, 1.8609906400E-001,
X 5.76645910421E-002, 3.7993059132E-002, 1.713387657E-002,
X 6.9444444448E-002, 1.0000000000E+000

DIMENSION APSV(21) $ DATA (APSV=-1.8643931093E+10,
X -1.8635237894E+009,-2.1829342088E+008,-2.5697908389E+007,
X -3.2145365220E+006,-4.2895240048E+005,-6.1335706678E+004,
X -9.4463548250E+003,-1.5763573037E+003,-2.8703323717E+002,
X -5.750830524E+001,-1.8280729308E+001,-3.2104935853E+000,
X -9.2047999257E-001,-3.0825376496E-001,-1.2410589605E-001,
X -6.2662163500E-002,-4.2462830794E-002,-4.3885030868E-002,
X -9.7222222227E-002. 1.0000000000E+000)

DIMENSION NQTT(15),NQTT(8) $ EQUIVALENCE (NQTT(8),NOT(1))
DATA(NQTT=1,3,7,12,17,23,29,35,41,47,53,59,64,68,71)
ANMIZI=ABSF(REAL(Z))+ABSF(IMAG(Z))
ENTRY AI $ LA=0 $ GO TO 1
ENTRY AIP $ LA=0 $ GO TO 2
ENTRY W1 $ ENTRY W1 $ LA=1 $ GO TO 1
ENTRY WIP $ ENTRY W1P $ LA=1 $ GO TO 2
ENTRY W2 $ LA=-1 $ GO TO 1
ENTRY W2P $ LA=-1 $ GO TO 2

1 LB=0 $ GO TO 3
2 LB=1 $ GO TO 3
3 Z=Z
4 IF(LA) 5,7,4
5 U=(-0.5,0.86602540378) $ GO TO 6
6 Z=U$Z
7 LC=0 $ IF(X(2)) 8,9,10
8 LC=1 $ X(2)=-X(2) $ GO TO 10
9 X(2)=0.
10 CONTINUE COMPARE WITH PREVIOUS
IF(X(1) NE. X(1)) OR. X(2) NE. X(2)) GO TO 20
1 I=LB+1 $ I=LG AND I $ IF(I) GO TO 400
IF(LB) 220,210
400 CONTINUE EXIT
IF(LB) 402,401
401 ZT=A $ IF(LC)XT(2)=XT(2)
IF(LA) 404,411,403
402 ZT=AP $ IF(LC)XT(2)=-XT(2)
IF(LA) 403,411,404
403 U=(1.,-1.7320508076) $ GO TO 410
404 U=(1.,1.7320508076) $ GO TO 410
410 ZT=U*ZT
411 AIRY=ZT $ RETURN

20 CONTINUE AFFINE COORDINATES
M=0
Z1=Z $ LG=0
IF(X(1))LE.-7. OR. X(1)GT.7. OR. X(2)GT.6.928203232) GO TO 200
IP=7-X(1) $ P=IP=7-IP
Q=IQ=0.86602540378*X(2)+0.5*(P-X(1))
N=NOT(IP)+IQ
IF(N.GE.NOT(IP+1)) GO TO 200

100 CONTINUE SERIES
XT(1)=P $ XT(2)=1.1547005384*Q
A=B2+B1 $ AP=AP+B3
AN=1.
DO 110 I=2,3
AN=AN+1.
B3=B3/U/AN
A=B3+A
B0=B1 $ B1=B2 $ B2=B3
B3=(ZT+B1+U*B0)*U/AN
AP=B3+AP
IF(ANM(B2).GT.0.5E-10*ANM(A) .AND. ANM(B3).GT.0.5E-10*ANM(AP)) I=0

110 CONTINUE
LG=3
GO TO 400

200 CONTINUE ASYMPTOTICS
ZA=CSQRT(Z)
ZB=0.28209479177/CSQRT(ZA)
ZT=-0.6666666666666666*Z*ZA
T=XT(1)**2+XT(2)**2
CALL ZEXP(XT(1),XT(2),SX,SY,M)
ZE=CMPLX(SX,SY)
ZM=2.718281828**(M+M1)
ZR=1./ZT
IF(XT(2) .GT. 0. .AND. XT(1) .LT. 11.8595) LG=4
DO 201 NT=2,18
IF(T .LT. ASLT(NT-1)) GO TO 202

201 CONTINUE $ NT=19
202 IF(LB) 220,210
210 CONTINUE A
ZT=ASV(NT-1)
DO 211 I=NT+21
211 ZT=ASV(I)+ZT*ZR
A=ZT*ZE
I=4.*AND.*LG $ IF(I) 212,216
212 ZT=ASV(NT-1)
DO 213 I=NT+21
213 ZT=ASV(I)-ZT*ZR
A=A+(0.1.*ZT/(ZE*ZM))
216 A=ZB*A $ LG=1.OR.LG $ GO TO 401
220 CONTINUE AP
ZT=APSV(NT-1)
DO 221 I=NT+21
221 ZT=APSV(I)+ZT*ZR
AP=ZT*ZE
I=4.*AND.*LG $ IF(I) 222,226
222 ZT=APSV(NT-1)
DO 223 I=NT+21
223 ZT=APSV(I)-ZT*ZR
AP=AP+(0.1.*ZT/(ZE*ZM))
226 AP=2*A*ZB*AP $ LG=2. OR.LG $ GO TO 402
END
7. APPENDIX: AN IONOSPHERIC REFLECTION COEFFICIENT PROGRAM

7.1 General Description

The propagation program described in the main body of this report requires ionospheric reflection coefficients as input. We will now describe briefly a computer program that can be used to calculate such reflection coefficients for any profile of ionospheric electron density.

We assume a plane ionosphere whose electron density and electron-neutral collision frequency vary only with altitude and a static magnetic field with constant dip angle and magnitude. For an infinite plane wave incident from below on the ionosphere at an angle, $\phi$, we compute the reflection coefficient matrix defined by

$$\begin{bmatrix} r_e & r_m \\ e & e \\ m & m \end{bmatrix} = \begin{bmatrix} T_{ee} & T_{em} \\ T_{me} & T_{mm} \end{bmatrix} \begin{bmatrix} E_e \\ i \\ E_m \end{bmatrix}$$

(A1)

where the pre-subscript indicates the incident or reflected wave and the post-subscript indicates whether the electric or magnetic vector is in the plane of incidence (e is "vertical polarization," m is "horizontal polarization").

The solution is accomplished by (1) dividing the ionosphere into thin homogeneous slabs whose properties are those of the continuous profile at the center of the slab, (2) solving Maxwell's equations in each slab, and (3) satisfying the boundary conditions at the slab interfaces. The solution is derived by Johler and Harper (1962).

The user must determine the height profile of electron density, the magnetic field parameters (strength, dip angle, azimuth relative to direction of propagation), the radio frequency, and the angle of incidence. By modifying the appropriate subroutine, he may specify the collision frequency.
The magnetic field parameters are well known (certainly to the precision required for this application), and maps of them can be found for instance in Davies (1965).

The collision frequency is also fairly well known—it decreases almost exponentially with height with a slope that the experts agree on. There is about a factor of 3 disagreement on the absolute magnitude at a given height (or an uncertainty of about 3 km in the height for a given value). The collision frequency computed in the subroutine CFEO is that recommended by Gambill and Knapp (1969). You can change subroutine CFEO to return any collision frequency profile you want.

We do not know the height distribution of electrons in the lower ionosphere very well yet; nor do we know how the electron density varies with season, sunspot cycle, latitude, etc. So we have included two versions of subroutine ENN to define electron density as a function of height. One simply interpolates in a table supplied as input by the user. The other contains a crude model ionosphere, which varies with the sun's zenith angle. This latter model (described by Berry and Jones (1970)) does a fair job of predicting the LF and VLF field strength variations during the day, especially at middle latitudes. You can easily incorporate any model of the ionosphere you want into the calculation by writing an appropriate version of ENN and substituting it for the version listed in this report.

Figure 14 shows a large block diagram of the organization of the program ANIREF. Details of the implementation can be determined from the comments cards in the FORTRAN listing at the end of this appendix.

7.2 A Sample Case

Table 2 lists the input parameters required by the program. The input for a sample case is shown in figure 15. The block diagram in
Start

22
Read frequency set
Read magnetic field parameter set
Read angles of incidence

30
Read an electron density profile

If blank go to new case

Begin frequency loop

Begin magnetic parameter loop

Begin angle of incidence loop

Compute reflection coefficients, print and punch

280
Last angle of incidence?

no
change angle

yes

290
Last magnetic parameter?

no
change mag. para.

yes

300
Last frequency?

no
change frequency

yes

Figure 14. Flow chart for Program ANIREF.
### Table 2. Input Data for Program ANIREF

<table>
<thead>
<tr>
<th>Variable</th>
<th>Physical Quantity</th>
<th>Units</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>IFR</td>
<td>Number</td>
<td></td>
<td>Number of frequencies to be read in; maximum of 9.</td>
</tr>
<tr>
<td>FRE(I)</td>
<td>Radio frequency</td>
<td>kHz</td>
<td></td>
</tr>
<tr>
<td>NPA</td>
<td>Number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HA(I)</td>
<td>Angle</td>
<td>Degrees</td>
<td>Azimuth of propagation of incident wave measured clockwise from north.</td>
</tr>
<tr>
<td>PHA(I)</td>
<td>Angle</td>
<td>Degrees</td>
<td>Earth's magnetic field intensity.</td>
</tr>
<tr>
<td>HAP(I)</td>
<td>Magnetic strength</td>
<td>Gauss</td>
<td></td>
</tr>
<tr>
<td>DAP(I)</td>
<td>Dip</td>
<td>Degrees</td>
<td>Magnetic field dip angle, measured from horizontal.</td>
</tr>
<tr>
<td>NPI</td>
<td>Number</td>
<td></td>
<td>Number of angles of incidence; maximum of 10.</td>
</tr>
<tr>
<td>SPHI(I)</td>
<td>Angle</td>
<td>Degrees</td>
<td>Angle of incidence of radio wave on ionosphere; measured from vertical.</td>
</tr>
<tr>
<td>PROFILE(I)</td>
<td>Alphanumeric identification of ionospheric model.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HITE</td>
<td>Height</td>
<td>km</td>
<td>Height of bottom of ionosphere. Should equal Z(L) if ionosphere is defined by a table.</td>
</tr>
<tr>
<td>LAYR</td>
<td>Number</td>
<td></td>
<td>Number of data cards defining ionospheric model.</td>
</tr>
<tr>
<td>Z(L)</td>
<td>Height</td>
<td>km</td>
<td>Usually a table defining electron density as a function of height. Can also be used to define parameters of an analytic model of electron density.</td>
</tr>
<tr>
<td>EN(L)</td>
<td>Electron density</td>
<td>electrons per cc</td>
<td></td>
</tr>
</tbody>
</table>
Figure 15. List of input data deck for ANIREF sample case.
figure 16 shows possibilities of data deck set up when several cases are to be run at one time.

The printed output for this sample case is listed in figures 17 and 18. The punched cards produced by this sample case are listed in figure 19. These cards are in the right format to be used as input to ANIHOP and were used as input for the sample case in the main body of this report (compare fig. 4).

The input electron density profile is printed on the first page of output (fig. 17). Then a table of computed reflection coefficients is printed under a heading which contains the other relevant input information. Comments have been superimposed on the printout to help explain the various entries.

7.3 Mathematical Details of Reflection Coefficient Calculation

The main subroutine of ANIREF solves the set of equations
Figure 16. Data deck setup for Program ANIREF.
Figure 17. Printed output from ANIREF sample case, page 1.
<table>
<thead>
<tr>
<th>FREQ</th>
<th>PHI</th>
<th>TEE AMPL</th>
<th>TEE PHASE</th>
<th>TEM AMPL</th>
<th>TEM PHASE</th>
<th>TME AMPL</th>
<th>TME PHASE</th>
<th>TMM AMPL</th>
<th>TMM PHASE</th>
<th>HT</th>
<th>DELTA Z</th>
<th>LAMBDA</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.00</td>
<td>65.00</td>
<td>0.23656519</td>
<td>1.02661192</td>
<td>0.18302232</td>
<td>-1.01805620</td>
<td>0.07601212</td>
<td>1.04136447</td>
<td>0.13321719</td>
<td>-1.04922457</td>
<td>76.3</td>
<td>1408</td>
<td></td>
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<tr>
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<td>0.49378107</td>
<td>0.56485196</td>
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<tr>
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<td>0.15273983</td>
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<td>0.19223038</td>
<td>0.05918210</td>
<td>-0.79775953</td>
<td>0.90805832</td>
<td>1.43249994</td>
<td>76.2</td>
<td>1408</td>
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<td></td>
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<tr>
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<td>0.20111441</td>
<td>0.47479994</td>
<td>0.21552332</td>
<td>0.23087025</td>
<td>0.12709565</td>
<td>0.44536462</td>
<td>0.65297111</td>
<td>1.71344939</td>
<td>77.1</td>
<td>1408</td>
<td></td>
</tr>
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<td>0.20210712</td>
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<td></td>
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<td>0.20041955</td>
<td>0.12547456</td>
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<td>1413</td>
<td></td>
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<td></td>
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<td>0.52889414</td>
<td>0.22236194</td>
<td>-1.14195296</td>
<td>0.23429944</td>
<td>-0.82783711</td>
<td>0.20806923</td>
<td>0.63034127</td>
<td>77.6</td>
<td>1386</td>
<td></td>
</tr>
<tr>
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<td>1.14126944</td>
<td>0.26631945</td>
<td>-1.14195296</td>
<td>0.23429944</td>
<td>-0.82783711</td>
<td>0.20806923</td>
<td>0.63034127</td>
<td>77.6</td>
<td>1386</td>
<td></td>
</tr>
<tr>
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<td>0.99777770</td>
<td>0.40373975</td>
<td>1.67116366</td>
<td>0.35962760</td>
<td>0.32105013</td>
<td>1.91263570</td>
<td>74.6</td>
<td>1408</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>106.00</td>
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<td>0.67164656</td>
<td>0.39756767</td>
<td>2.01668815</td>
<td>0.35328167</td>
<td>0.39052596</td>
<td>1.72956226</td>
<td>76.2</td>
<td>1408</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.00</td>
<td>108.00</td>
<td>0.14643622</td>
<td>0.49519245</td>
<td>0.39022334</td>
<td>2.18509825</td>
<td>0.36246033</td>
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<td>0.18273117</td>
<td>0.40046964</td>
<td>0.37914027</td>
<td>2.35607602</td>
<td>0.33597971</td>
<td>2.29893440</td>
<td>0.47287972</td>
<td>1.97351555</td>
<td>78.2</td>
<td>1392</td>
<td></td>
</tr>
<tr>
<td>30.00</td>
<td>112.00</td>
<td>0.18859483</td>
<td>1.98731976</td>
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<td>0.37545620</td>
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<td>114.00</td>
<td>0.19222218</td>
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<td>2.34473675</td>
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</tr>
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<td>30.00</td>
<td>116.00</td>
<td>0.23159478</td>
<td>2.19860970</td>
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<td>2.53246226</td>
<td>0.34091375</td>
<td>2.59743933</td>
<td>0.47817761</td>
<td>0.93301888</td>
<td>79.4</td>
<td>1397</td>
<td></td>
</tr>
</tbody>
</table>

Figure 18. Printed output from ANIREF sample case, page 2.
<table>
<thead>
<tr>
<th>Identification</th>
<th>Azimuth</th>
<th>Dip Angle</th>
<th>Field Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>SAN FRAN SEAPATH NIGHT</td>
<td>PHIA = 213.5</td>
<td>DIP = 6.5</td>
<td>HM = 0.310 GAUSS</td>
</tr>
<tr>
<td>30065.02386-001</td>
<td>1.9271</td>
<td>830-001</td>
<td>1.0187</td>
</tr>
<tr>
<td>30073.01627-001</td>
<td>2.8252</td>
<td>093-001</td>
<td>0.9357</td>
</tr>
<tr>
<td>30078.01527-001</td>
<td>2.5762</td>
<td>187-001</td>
<td>1.8299</td>
</tr>
<tr>
<td>30080.02031-001</td>
<td>2.4752</td>
<td>135-001</td>
<td>2.2311</td>
</tr>
<tr>
<td>30081.02410-001</td>
<td>2.4752</td>
<td>081-001</td>
<td>2.3921</td>
</tr>
<tr>
<td>30082.02866-001</td>
<td>2.5042</td>
<td>005-001</td>
<td>2.5481</td>
</tr>
<tr>
<td>30065.03816-001</td>
<td>1.1412</td>
<td>662-001</td>
<td>1.1152</td>
</tr>
<tr>
<td>30073.02596-001</td>
<td>2.6713</td>
<td>786-001</td>
<td>0.7353</td>
</tr>
<tr>
<td>30078.01394-001</td>
<td>2.9984</td>
<td>035-001</td>
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<td>30080.01270-001</td>
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<td>977-001</td>
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</tr>
<tr>
<td>30081.01464-001</td>
<td>2.4953</td>
<td>902-001</td>
<td>2.1853</td>
</tr>
<tr>
<td>30082.01827-001</td>
<td>2.4003</td>
<td>791-001</td>
<td>2.3513</td>
</tr>
</tbody>
</table>

Figure 19. List of punched card output from ANIREF sample case.
The elements of the matrix are defined by Johler and Harper (1962) in their equations (33-35) (note an error in the general form of their subscripts). They depend on the roots of a quartic equation (8 through 20 of Johler and Harper (1962)), which in turn depends on the ionospheric model, radio frequency, and direction of propagation.

Instead of solving (A2) directly, we divide the matrix into block matrices:

\[
\begin{pmatrix}
A_{11} & A_{12} & 0 & 0 \\
A_{21} & A_{22} & A_{23} & 0 \\
0 & A_{32} & A_{33} & A_{34} \\
0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
U_1 \\
U_2 \\
U_3 \\
U_4
\end{pmatrix}
= 
\begin{pmatrix}
K \\
0 \\
0 \\
0
\end{pmatrix}
\]

(A3)

Consider the rows as separate equations and solve the last one for \( U_p \)

\[
U_p = -A^{-1}_{pp} A_{p-1,p-1} U_{p-1}.
\]

(A4)

Substitute this value into the next-to-last equation, so it has only two unknowns and solve for \( U_{p-1} \). Repeat the process until \( U_1 \) is found.

The algorithm is: Let

\[
B_p = A_{pp}
\]

(A5)

and let

\[
B_k = A_{kk} - A_{k,k+1} B_{k+1}^{-1} A_{k+1,k'} \text{ for } 1 \leq k < p.
\]

(A6)

Then

72
\[ B_1 U_1 = K, \text{ or } U_1 = B_1^{-1} K. \quad (A7) \]

Since only columns 3 and 4 of \( A_{k+1,k} \) and columns 1 and 2 of \( A_{k,k+1} \) are nonzero, the elements of the \( A_{ij} \) are stored in a \( 4p \times 8 \) array arranged as follows:

<table>
<thead>
<tr>
<th>Column No.</th>
<th>1,2</th>
<th>3,4,5,6</th>
<th>7,8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( A_{11} )</td>
<td>( A_{12} )</td>
<td></td>
</tr>
<tr>
<td>( A_{21} )</td>
<td>( A_{22} )</td>
<td>( A_{23} )</td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td></td>
</tr>
<tr>
<td>( A_{p,p-1} )</td>
<td>( A_{pp} )</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

The real and imaginary parts of (A8) are stored in separate arrays. The rows \( a_{ik} \) and \( b_{ik} \) of each layer (see (A2)) have been interchanged for convenience. If the elements of (A8) are denoted \( \alpha_{ij} \), then

\[
\alpha_{4p+1,j} = b_{p,m}, \quad \alpha_{4p+2,j} = a_{p,m}, \quad (A9)
\]

\[
\alpha_{4p+3,j} = c_{p,m}, \text{ and } \alpha_{4p+4,j} = d_{p,m},
\]

where \( m = 4p + j - 6 \), and \( a_{ij}, b_{ij}, c_{ij}, \text{ and } d_{ij} \) are the elements of (A2).

After the array (A8) has been generated, the \( B_k \) are computed as follows:

1. \( B_k^{-1} A_{k,k-1} \) is computed and stored in \( A_{kk} \). \( B_k^{-1} \) is found by solving \( B_k B_k^{-1} = I \).
2. Then \( B_{k-1} \) is calculated using (A6) and stored in \( A_{k-1,k-1}' \) and the process is repeated.
When $B_1$ has been calculated, columns 1 and 2 are interchanged with columns 3 and 4 (to prevent zeros on the diagonal in certain cases) and (A7) is solved. The reflection coefficients have now been found.

The phase of the reflection coefficient is referenced to the bottom of the ionospheric profile. We also compute the quasi-stationary phase height of $T_{ee}$ as follows: There is a height where the phase of $T_{ee}$ is nearly $\pi$ over a considerable range of angles of incidence near grazing. In general the phase of $T_{ee}$ at height $h_r$ in terms of the phase at $h_o$ is

$$p_r = p_o + \frac{4\pi}{\lambda} \cos \psi (h_r - h_o),$$

(A10)

where $p_i$ is the phase at height $h_i$. So let $p_r = \pi$ and solve for $h_r$:

$$h_r = h_o + \frac{\pi - p_o}{\frac{4\pi \cos \psi}{\lambda}}.$$

(A11)

We call $h_r$ the quasi-stationary phase height.
7.4 FORTRAN Listing of Program ANIREF and Its Subroutines

PROGRAM ANIREF

COMPUTES IONOSPHERIC REFLECTION COEFFICIENTS FOR STRATIFIED IONOSPHERE
BY L. A. BERRY AND J. E. HERMAN, OFFICE OF TELECOMMUNICATIONS
REFERENCE IS TELECOMMUNICATIONS RESEARCH REPORT NO. 11, ITS BOULDER COLORADO, 80302

DIMENSION FRE(9), SPHI(10), PHA(8), HA(8), DAP(8)
DIMENSION PROFILE(5), EN(80), Z(80)
DIMENSION PR(4), PI(4), QR(4), QI(4), ZR(4), ZI(4)
DIMENSION AR(600, 8), AI(600, 8)
DIMENSION ROOT(8), COF(10)
COMMON AR, AI, PR, PI, QR, QI, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1 CI, SI, HM, OMEGA
2 HA, MA, WAVE, COF
COMMON/1/ Z, EN, LAYR, TEER, TEEI, TEEA, TEEP, TEMR, TEMI, TEMA, TEMP,
1 TEMR, TEMI, TMEA, TMEP, TMMR, TMMI, TMMMA, TMMMP
1 FORMAT(F10.2, E10.2)
2 FORMAT(1H1)
3 FORMAT(* FREQ PHII TEE AMPL TEE PHASE TEM AMPL TE
1M PHASE TME AMPL TME PHASE TMM AMPL TMM PHASE HT
2 KLOCK *)
4 FORMAT(1H1)
6 FORMAT(2F8.2, 12X)
7 FORMAT(5A6, 5H H = F5.2, 5H KM, 15, 8H POINTS.)
8 FORMAT(3F10.0)
9 FORMAT(1H0, 9X, 12HPHI SUB A = F7.2, 11H DEG., DIP = F7.2,
110H DEG., HM = F6.3, 18H GAUSS DELTA Z=F5.2, 7H LAMBDA )
19 FORMAT(12)
25 FORMAT(3F10.0)

DATA (RAT=.1)
22 READ 19, IFR

IFR IS NUMBER OF FREQUENCIES

IF (EOF, 60, 400, 20)
20 READ 8, (FRE(I), I=1, IFR)

FRE IS THE FREQUENCY IN KHZ

READ 19, NPA
NPA IS NUMBER OF MAGNETIC FIELD CONDITIONS

READ 25, (PHA(I), HA(I), DAP(I), I=1, NPA)
PHA = PHI A = MAGNETIC AZIMUTH
HA = HM = MAGNETIC FIELD STRENGTH IN GAUSS
DAP = DIP = MAGNETIC FIELD DIP ANGLE IN DEGREES
CAUTION—- (DIP=0) AND (PHIA=90 OR 270) IS NOT ALLOWED

READ 19, NPI
NPI = NUMBER OF ANGLES OF INCIDENCE, PHI I, IN DEGREES

75
READ 8, (SPHI(I), I=1, NPI)
30 READ 7, (PROFILE(I), I=1, 5), HITE, LAYR
C PROFILE IS ALPHANUMERIC DESCRIBING IONO PROFILE, HITE IS HT OF BOTTOM OF IONOS IN KM, LAYR IS THE NUMBER OF DATA CARDS IN IONO PROFILE TO FOLLOW
C A BLANK CARD RETURNS THE PROGRAM TO 22 TO READ A NEW SET OF PARAMETERS
C
IF(EOF, 60) 400, 31
31 CONTINUE
IF (LAYR) 22, 22, 32
32 PRINT 4
PRINT 4
PRINT 7, (PROFILE(I), I=1, 5), HITE, LAYR
DO 33 L=1, LAYR
READ 1, (L), EN(L)
C Z(L) IS THE HEIGHT IN KM, EN(L) IS THE ELECTRONS/CC AT HT Z(L)
C
PRINT 1, Z(L), EN(L)
33 CONTINUE
PRINT 4
PRINT 7, (PROFILE(I), I=1, 5), HITE, LAYR
C BEGINNING OF FREQUENCY LOOP
C
DO 300 J=1, IFR
F=FRE(J)
DELTA = RAT*300/F
C
C BEGINNING OF MAGNETIC PARAMETERS LOOP
C
DO 290 K=1, NPA
PHIA=PHA(K) $ HM=HA(K) $ DIP=DAP(K)
PRINT 9, PHIA, DIP, HM, RAT
LINE =0
PRINT 2
PRINT 2
PRINT 3
RDG= .017 453 292 519 943
FIA=RDG*PHIA
DD= RDG*DIP
CFIA=COSF(FIA)
SFIA=SINF(FIA)
SI =SINF(DD)
CI =COSF(DD)
100 CONTINUE
PUNCH 77, (PROFILE(I), I=1, 5), PHIA, DIP, HM
C
C BEGINNING OF ANGLE OF INCIDENCE LOOP
C
DO 280 L=1, NPI
PHII=SPHI(L)
110 FII=RDG*PHII
CFII = COSF(FII) MOD UP
SFII = SINF(FII) MOD UP
AT = SFII * SFIA MOD UP
AL = SFII * CFIA MOD UP
K1 = KLOCK(0)
150 CALL REFCOF
C REFCOF COMPUTES THE REFLECTION COEFFICIENTS. INPUT AND OUTPUT ARE IN
C COMMON/1/
C
160 K2 = KLOCK(1)
C COMPUTE QUASI STATIONARY PHASE HEIGHT
HREF = HITE + (3.1416 - TEEP) / (2 * WAVE * CFII)
PRINT 6, F, PHI, TEEA, TEEP, TEMA, TEMP, TMEP, TMMA, TMM
1, HREF, K2
PUNCH 1111, F, PHI, TEEA, TEEP, TEMA, TEMP, TMEP, TMMA, TMM, HITE, 1
HREF
1111 FORMAT (F6.1, F4.1, 4(E9.3, F6.3), 2F5.1)
170 LINE = LINE + 1
IF (LINE .LE. 50) GO TO 200
PRINT 4
PRINT 7, (PROFILE(I), I = 1, 5), HITE, LAYR
PRINT 9, PHIA, DIP, HM, RAT
LINE = 0
PRINT 2
PRINT 3
PRINT 2
200 CONTINUE
C END OF ANGLE OF INCIDENCE LOOP
C
280 CONTINUE
290 CONTINUE
C END OF FREQUENCY LOOP
300 CONTINUE
GO TO 30
400 CALL EXIT
END
SUBROUTINE REFCOF

MAIN SUBROUTINE FOR ANIREF, COMPUTES REFLECTION COEFFICIENTS.
REFERENCE IS TELECOMMUNICATIONS RESEARCH REPORT NO. 11,
BY L. A. BERRY AND J. E. HERMAN, OFFICE OF TELECOMMUNICATIONS
ITS, BOULDER COLORADO, 80302

COMMON AR, AI, PR, P1, QR, OI, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1 CI, SI, HM, OMEGA
2 ZMA, WAVE, COF
COMMON/1/ Z, EN, LAYR, TEER, TEE, TEEA, TEEP, TEMR, TEMI, TEMA, TEMP,
1 TME, TMEI, TMEA, TMEP, TMAR, TMMI, TMMA, TMMR
COMMON/4/ BAR, BAI
DIMENSION PR(4), PI(4), OR(4), OI(4),
1 DIMENSION BAR(4,8), BAI(4,8), COF(10), ROOT(8), P(8), Q(8)
DIMENSION AR(6100,8), AI(6000,8), Z(80), EN(80)
OMEGA=6283.1853*F
WAVE=OMEGA/2.997925E5

NLAY = (Z(LAYR) - Z(1)) / DELTA

C THERE ARE NLAY SLABS

IF (NLAY .LE. 149) GO TO 10
NLAY = 149
DEL = (Z(LAYR) - Z(1)) / 149.
PRINT 9, DEL
9 FORMAT (* CHANGED DELTA Z TO *, F6.2, * LAMBDAA*)

C ISA=CFII*SFIA
C DEL=1.1415927/(2.*WAVE)
C ICA=CFII*CFIA

C INITIAL GUESS FOR ROOTS OF BOOKER QUARTIC AT BOTTOM OF IONOSPHERE

ROOT(1)=CFII
ROOT(2)=0.
ROOT(3)=CFII
ROOT(4)=0.
ROOT(5)=-CFII
ROOT(6)=0.
ROOT(7)=-CFII
ROOT(8)=0.
ZK=WAVE*DELTA
ZM=Z(1)+5*DELTA
ELEC=ENN(ZM)
COL=CFEO(ZM)

C ZM IS HEIGHT AT MIDDLE OF SLAB
C ELEC IS ELECTRON DENSITY OF SLAB, COL IS COLLISION FREQUENCY OF SLAB

CALL QPZ(ELEC, COL)
CALL FOLEST(8,1)

C THE CALL TO Q9EXUN SURPRESSES THE SYSTEM EXPONENT UNDERFLOW ERROR
CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

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CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN

CALL Q9EXUN
DO 140 K=2,4
LPK=LP+K
TX=BAR(I,K+4)*AR(LPK,JA)-BAI(I,K+4)*AI(LPK,JA)
TY=BAI(I,K+4)*AR(LPK,JA)+BAR(I,K+4)*AI(LPK,JA)
AR(LPA,J)=AR(LPA,J)+TX
140 AI(LPA,J)=AI(LPA,J)+TY

THIS LOOP IS STEP 2, FOLLOWING Eq(A9)

DO 150 I=1,4
LPA=LP-4+I
DO 150 J=5,6
JA=J+2
TX=AR(LPA,7)*AR(LP+1,J)-AI(LPA,7)*AI(LP+1,J)
TY=AI(LPA,7)*AR(LP+1,J)+AR(LPA,7)*AI(LP+1,J)
SX=AR(LPA,8)*AR(LP+2,J)-AI(LPA,8)*AI(LP+2,J)
SY=AI(LPA,8)*AR(LP+2,J)+AR(LPA,8)*AI(LP+2,J)
AR(LPA,J)=AR(LPA,J)-TX-SX
150 AI(LPA,J)=AI(LPA,J)-TY-SY

DO 95 J=1,2
DO 95 I=1,4
BAR(I,J)=AR(I,J+4)
BAI(I,J+2)=0.
95 BAI(I,J)=AI(I,J+4)
BAR(1,3)=SFIA
BAR(1,4)=CICA
BAR(2,3)=-CFIA
BAR(2,4)=CISA
BAR(3,3)=CISA
BAR(3,4)=CFIA
BAR(4,3)=CICA
BAR(4,4)=-SFIA

CALL MINVERT
TEMR=BAR(3,5)*CICA+BAR(3,6)*CISA-BAR(3,7)*AI(3,8)*SFIA
TEMI=BAR(3,5)*CICA-BAI(3,6)*CISA-BAI(3,7)*AI(3,8)*SFIA
TEER=BAR(4,5)*CICA+BAR(4,6)*CISA-BAR(4,7)*AI(4,8)*SFIA
TEEI=BAR(4,5)*CICA+BAI(4,6)*CISA-BAI(4,7)*AI(4,8)*SFIA
TMMR=-BAR(3,5)*SFIA+BAR(3,6)*CFIA+BAR(3,7)*CISA+BAR(3,8)*CICA
TMMI=-BAI(3,5)*SFIA+BAI(3,6)*CFIA+BAI(3,7)*CISA+BAI(3,8)*CICA
TMER=-BAR(4,5)*SFIA+BAR(4,6)*CFIA+BAR(4,7)*CISA+BAR(4,8)*CICA
TMEI=-BAI(4,5)*SFIA+BAI(4,6)*CFIA+BAI(4,7)*CISA+BAI(4,8)*CICA
TEEA=CABS(CMPLX(TEER,TEEI))
TEEP=CANG(CMPLX(TEER,TEEI))
TEMA=CABS(CMPLX(TEMR,TEMI))
TEMP=CANG(CMPLX(TEMR,TEMI))
TMEA=CABS(CMPLX(TMER,TMEI))
TMEE=CANG(CMPLX(TMER,TMEI))
TMMA=CABS(CMPLX(TMRR,TMMI))
TMMP=CANG(CMPLX(TMRR,TMMI))
RETURN
FUNCTION EEN(ZM)
C USES LOGARITHMIC INTERPOLATION IN TABLE OF N(H) TO GET ELECTRON DENSITY AT ZM

DIMENSION Z(80), EN(80)
COMMON/1/ Z*EN*LAYR*TEER*TEEI*TEEAA*TEEP*TEM*TEMI*TEEA*TEM*TEMP,
1 TMER*TMEE*I*TMMP*TMMM*TMWI*TMMA*TMMP
KID=1
IF(Z<LAYR)-ZM>70,70,25
70 ENN=EN<LAYR>1
GO TO 71
25 IF(Z<KID+1)-ZM>26,27,28
26 KID=KID+1
GO TO 25
27 ENN=EN<KID+1>
GO TO 71
28 TX=(ZM-Z<KID1))/(Z<KID+1>-Z<KID1>)
ENN= EN<KID1>*EXP1TXLOGF(EN<KID+1)/EN<KID1>)
71 CONTINUE
RETURN
END

FUNCTION CFEO(H)
C DASA COLLISION FREQUENCY MODEL FIT WITH EXP(P9),
WHERE P9 IS A NINTH DEGREE POLYNOMIAL.

DIMENSION A(10)
DATA((A(I), I=1,9 )=2.587803463E1, -1.210027715E-1, -1.462645167E-3
1, -1.172264046E-5, 1.749042668E-6, -2.948406644E-8, 1.351055095E-10,
4.11118378E-13, -3.289391577E-15 )
SUM =A(9)
DO 10 J=1,8
10 SUM = H*SUM +A(9-J)
CFEO = EXPF(SUM)
RETURN
END
SUBROUTINE CQPZ(ELEC, COL)

SUBROUTINE FOR REFCOFe RETURNS THE ROOTS OF THE QUARTIC, AND THE
FIELD RATIOS P AND Q, SEE JOHLER AND HARPER, JAN 1961 RADIO SCIENCE.

COMMON AR, AI, PR, PI, QR, QI, ROOT, AL, AT, F, DELTA, CFIA, SFIA,
1 CI, SI, HM, OMEGA
2 *ZMA, WAVE, COF
DIMENSION COF(10), PR(4), PI(4), QR(4), QI(4), ZR(4), ZI(4)
DIMENSION AR(600,8), AI(600,8)
DIMENSION ROOT(8), TOOT(4), ZOOTR(4), ZOOTI(4), ZZR(4), ZZI(4)

JU=1
JD=4
Y=1.75888E7*HM/OMEGA
YT= Y*C1
YL= Y*SI
X=(3.1824858E9*ELEC)/(OMEGA*OMEGA)
Z=+COL/OMEGA
ATSQ=AT*AT
ALSQ=AL*AL
CSQ =1. -ATSQ-ALSQ
XSQ=X*X
YSQ =Y*Y
ZSQ=Z*Z
YLSQ=YL*YL
YTSQ=YT*YT
UR=1.
UI=-Z
U2R=1. -ZSQ
U2I=-2. *Z
TAR=1.-ZSQ-YSQ
TAI=2.*Z
TX=TAR*TAR+TAI*TAI
TX=-X/TX
TAR=TAR*TX
TAI=TAI*TX
PXXR=TAR -TAI*UI
PXXI=TAR*UI+TAI
PYXR=-YL*TAI
PYXI= YL* TAR
PZXR= YT*TAI
PZXI=-YT* TAR
PZXR=-PZXR
PZXI= PZXI
TX=1.+ZSQ
TBR=(TAR-TAI*Z)/TX
TBI=(TAI+TAR*Z)/TX
TCR=U2R-YLSQ
TCI=U2I
PZZR=TBR*TCR-TBI*TCI
PZZI=TBR*TCI+TBI*TCR
PYXR= -YL*YT*TBR
PYXI= YL*YT* TBI
TCR=U2R-YTSQ

82
PYYR = TBR * TCR - TBI * TCI
PYYI = TBR * TCI + TBI * TCR
TCR = PXR * (1 + ATSQ) + PYYR * (1 + ALSQ) + PZZR
TCI = PXXI * (1 + ATSQ) + PYYI * (1 + ALSQ) + PZZI

COEFFICIENTS OF BOOKER QUARTIC

COF(9) = 1 + PZZR
COF(10) = PZZI
COF(7) = 2 * AL * PZRY
COF(8) = 2 * AL * PZYI
COF(5) = -2 * CSQ + 2 * X * TAR - TCR - CSQ * PZZR
COF(6) = 2 * X * TAI - TCI - CSQ * PZZI
COF(3) = -CSQ * COF(7)
COF(4) = -CSQ * COF(8)
COF(1) = CSQ * CSQ + CSQ * TCR - (2 * CSQ + 1) * X * TAR + X * CSQ + TBR
COF(2) = -CSQ * CSQ - CSQ * TCI - (2 * CSQ + 1) * X * TAI + X * CSQ + TBI

CALL ZROOT
DO 9 I = 1, 4
II = I + 1
ZR(I) = ROOT(I + 1)
ZI(I) = ROOT(I + II)
9 CONTINUE
R12R = AL * AT + PXYR
R12I = PXYI
R33R = CSQ + PZZR
R33I = PZZI

COMPUTE P AND Q IN THE 40 LOOP

DO 40 I = 1, 4
ZRSQ = ZR(I) * ZR(I) - ZI(I) * ZI(I)
ZISQ = 2 * ZR(I) * ZI(I)
R11R = 1 - ALSQ - ZRSQ + PXXR
R11I = ZISQ + PXXI
R13R = AT * ZR(I) + PXXR
R13I = AT * ZI(I) + PXXI
R31R = AT * ZR(I) + PXXR
R31I = AT * ZI(I) + PXXI
R32R = AL * ZR(I) + PZRY
R32I = AL * ZI(I) + PZYI
DENI = R33I * R11I - R31R * R11I - R13R * R31I + R13R * R31I
TX = DENR * DENR + DENI * DENI
TX1 = PPR * DENR - PPI * DENI
PP1 = PPI * DENR + PPR * DENI
PPR = TX1
TX1 = QQR * DENR - QQ1 * DENI
QQ1 = QQ1 * DENR + QQR * DENI
QQR = TX1
SUBROUTINE FOLES(I,J)
C
SUBROUTINE FOR REFCOF. FILLS UP COL 5 - 8 OF EQ (A8).
C
DIMENSION PR(4), PI(4), QR(4), QI(4), ROOT(8), P(8), Q(8)
DIMENSION AR(600,8), AI(600,8)
DIMENSION COF(10)
COMMON AP, AI, PR, PI, QR, QI, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1 CI, SI, HM, OMEGA
2 *ZMA*WAVE*COF
DO 100 K=1,4
KK=2*K
P(KK-1) = PR(K)
P(KK) = PI(K)
Q(KK-1) = QR(K)
100 Q(KK) = QI(K)
DO 20 J=5,J
JP=2*J-9
AR(LP+J) = AL*P(JP-ROOT(JP))
AI(LP+J) = AL*P(JP+1)-ROOT(JP+1)
TX=ROOT(JP)*Q(JP)-ROOT(JP+1)*Q(JP+1)
TY=ROOT(JP+1)*Q(JP)+ROOT(JP)*Q(JP+1)
AR(LP+3,J) = TX-AT*P(JP)
20 AI(LP+3,J) = TY-AT*P(JP+1)
RETURN
END
SUBROUTINE ZROOT

ZROOT FINDS THE ROOTS OF A QUARTIC WITH COMPLEX COEFFICIENTS
USING THE NEWTON-RAPHSON METHOD TO FIND THE FIRST TWO ROOTS,
AND THE QUADRATIC FORMULA FOR THE OTHER TWO. COF(1)+I*COF(2) IS
THE CONSTANT TERM, COF(3)+I*COF(4) IS THE COEFFICIENT OF Z , ETC
THE ROOTS ARE ROOT(2*K-1)+I*ROOT(2*K), K=1,4 WRITTEN BY B. WEIDER

DIMENSION COF(10),ROOT(8)
DIMENSION AR(600,8), AI(600,8), VR(4), WI(4), WR(4), WI(4)
COMMON AR, AI, VR, WI, WR, ROOT, AL, AT, F, DELTA, CFII, SFIA, CFIA,
1 CI, SI, HM, OMEGA
2 ZMA, WAVE, COF
J=1
JRT=2
JCOF=10
1000 EPSILON=1.E-8
999 JROOT=JRT+JRT
L=0
C USE ROOT FOR PREVIOUS CASE AS FIRST GUESS
C
XR=ROOT(JROOT-1)
XI=ROOT(JROOT)
1 I=JCOF-J
L=L+1
YR=XR
YI=XI
FR=COF(I)
FI=COF(I+1)
2 I=I-2
TX=XR*FR-XI*FI+COF(I)
FI=XR*FI+XI*FR+COF(I+1)
FR=TX
IFCleGT.1l GO TO 2
A=FR
B=FI
C A+I*B=P(XR+I*XI)
C
I=JCOF-J
CCOF=(I-1)/2
GR=CCOF*COF(I)
GI=CCOF*COF(I+1)
4 I=I-2
CCOF=CCOF-1.
TX=XR*GR-XI*GI+CCOF*COF(I)
GI=XR*GI+XI*GR+CCOF*COF(I+1)
GR=TX
IFCleGT.3l GO TO 4
C=GR
D=GI
C C+I*D=DERIVATIVE OF P(XR+I*XI)
C
ASSIGN 5 TO LSW
GO TO 400
5 EPSR=X
EPSI=Y
C
EPSR*I*EPSI=CORRECTION TO ROOT.
C
A=EPSR
B=EPSI
C=XR
D=XI
ASSIGN 6 TO LSW
GO TO 400
6 X=ABSFX)
Y=ABSFY)
C
HAVE WE CONVERGED
C
IF(X.GT.EPSILON) GO TO 7
IF(Y.LT.EPSILON) GO TO 60
7 XR=XR-EPSSR
XI=XI-EPSI
IF(L.LE.10) GO TO 1
EPSILON=10.*EPSILON
L=0
IF(EPSILON.GT.1.E-6) PRINT 800,EPSILON,ZMA
800 FORMAT(1HO,** INCREASED EPSILON TO*,E9.1*, IN ZROOT AT HEIGHT*,
1 F8.2//)
GO TO 1
60 ROOT(JROOT-1)=XR
ROOT(JROOT)=XI
N=JCOF-3
61 A=COF(N+2)
B=COF(N+3)
X=A*XR-B*XI
Y=A*XI+B*XR
COF(N)=COF(N)+X
COF(N+1)=COF(N+1)+Y
N=N-2
IF(N.GT.1)GO TO 61
N=JCOF-2
DO 62 K=1,N
COF(K)=COF(K+2)
62 CONTINUE
JCOF=JCOF-2
JRT=JRT+2
IF(JRT.LE.4) GO TO 1000
C
FROM HERE TO 72+4 IS QUADRATIC FORMULA
C
X=COF(3)*COF(3)-COF(4)*COF(4)
A=COF(3)*COF(4)
Y=A*A
86
A = 4.0 * (COF(1) * COF(5) - COF(2) * COF(6))
B = 4.0 * (COF(1) * COF(6) + COF(2) * COF(5))
A = X - A
B = Y - B
ASSIGN 70 TO LSW
GO TO 500
70 A = X
B = Y
C = 2.0 * COF(5)
D = 2.0 * COF(6)
ASSIGN 71 TO LSW
GO TO 400
71 TX = X
TY = Y
A = COF(3)
B = COF(4)
ASSIGN 72 TO LSW
GO TO 400
72 A = -X + TX
B = -Y + TY
C = X - TX
D = Y - TY
IF(B) 73, 75, 74
73 ROOT(1) = A
ROOT(2) = B
ROOT(5) = C
ROOT(6) = D
GO TO 78
74 ROOT(1) = C
ROOT(2) = D
ROOT(5) = A
ROOT(6) = B
GO TO 78
75 IF(A GT 0.0) GO TO 76
ROOT(1) = C
ROOT(5) = A
GO TO 77
76 ROOT(1) = A
ROOT(5) = C
77 ROOT(2) = 0.0
ROOT(6) = 0.0
78 CONTINUE
RETURN

This is a complex division branch. X + iY = (A + iB)/(C + iD)

400 IF(C) 410, 405, 410
405 IF(D) 406, 440, 406
406 X = B / D
Y = A / D
GO TO LSW
410 IF(D) 415, 411, 415
411 X = A / C
Y = B / C

87
GO TO LSW
415 IF(ABSF(C)>ABSF(D)) 416, 430, 430
416 AX=C/D
   DEN=AX*C+D
   X=(A*AX+B)/DEN
   Y=(B*AX-A)/DEN
   GO TO LSW
430 AX=D/C
   DEN=C+AX*D
   X=(A+B*AX)/DEN
   Y=(B-A*AX)/DEN
   GO TO LSW
440 PRINT 401
401 FORMAT(* ZERO DIVISOR IN ZMROOT DIVIDE*)
   CALL EXIT
C
C THIS IS A COMPLEX SQUARE ROOT BRANCH. X+I*Y=SQRT(A+I*B).
C
C 500 S=1.*
   IF(B) 501, 505, 505
501 S=-1.*
505 IF(A) 502, 504, 502
502 X=SQRTF(1.5*(SQRTF(A*A+B*B)+ABSF(A)))
   D=B/(2.*X)
   Y=D
   IF(A*GE.0.) GO TO LSW
   Y=S*X
   X=S*D
   GO TO LSW
504 X=SQRTF(S*B*5)
   Y=S*X
   GO TO LSW
END
SUBROUTINE MINVERT

C INVERTS 4X4 COMPLEX MATRIX

COMMON /4/ BR,AR,BI, AI
DIMENSION IPVOT(4), INDEX(8), PIVOTR(4), PIVOTI(4)
DIMENSION AR(16), AI(16), BR(16), BI(16)
N=4 NSQ=16
DO 888 I=1,NSQ
AR(I)=BR(I)
AI(I)=BI(I)
888 CONTINUE
DETR=1.
DETI=0.
DO 17 J=1,N
IPVOT(J)=0
17 CONTINUE
DO 135 I=1,N
TR=0.
TI=0.
DO 9 J=1,N
IF(IPVOT(J)*EQ*1) GO TO 9
NK=N
DO 23 K=1,N
NK=NK+N
JK=J+NK
IF(IPVOT(K-1)*EQ*23,81,23)
43 IF((TR*TR+TI*TI)-(AR(JK)*AR(JK)+AI(JK)*AI(JK)))*EQ*23,23
83 IROW=J
ICOL=K
TR=AR(JK)
TI=AI(JK)
23 CONTINUE
9 CONTINUE
IPVOT(ICOL)=IPVOT(ICOL)+1
IF(IROW*EQ*ICOL) GO TO 109
DETR=-DETR
DETI=-DETI
NL=N
DO 12 L=1,N
NL=NL+N
IROWL=IROW+NL
ICOLL=ICOL+NL
TR=AR(IROWL)
TI=AI(IROWL)
AR(IROWL)=AR(ICOLL)
AI(IROWL)=AI(ICOLL)
AR(ICOLL)=TR
AI(ICOLL)=TI
12 CONTINUE
109 I1=1
I2=I+N
ICOL1ICOL=ICOL+N*(ICOL-1)
INDEX(I1)=IROW

89
INDEX(I2)=ICOL
PIVOTR(I)=AR(ICOL+ICOL)
PIVOTI(I)=AI(ICOL+ICOL)
TXR=DETR*PIVOTR(I)-DETI*PIVOTI(I)
TXI=DETR*PIVOTI(I)+DETI*PIVOTR(I)
DETR=TXR
DETI=TXI
AR(ICOL+ICOL)=1*
AI(ICOL+ICOL)=0*
NL=N
DO 205 L=1,N
NL=NL+N
ICOLL=ICOL+NL
DXR=PIVOTR(I)*PIVOTR(I)+PIVOTI(I)*PIVOTI(I)
TXR=AR(ICOLL)*PIVOTR(I)+AI(ICOLL)*PIVOTI(I)
TXI=AI(ICOLL)*PIVOTR(I)-AR(ICOLL)*PIVOTI(I)
AR(ICOLL)=TXR/DXR
AI(ICOLL)=TXI/DXR
205 CONTINUE
DO 135 LI=1,N
IF(LI-ICOL)21,135,21
LI=LI+N*(ICOL-1)
TR=AR(II+ICOL)
TI=AI(II+ICOL)
AR(II+ICOL)=0*
AI(II+ICOL)=0*
NL=N
DO 89 L=1,N
NL=NL+N
ICOLL=ICOL+NL
LIL=LI+NL
TXR=AR(ICOLL)*TR-AI(ICOLL)*TI
TXI=AI(ICOLL)*TR+AR(ICOLL)*TI
AR(LIL)=AR(LIL)-TXR
AI(LIL)=AI(LIL)-TXI
89 CONTINUE
135 CONTINUE
DO 3 I=1,N
L1=N-I+1
L2=L1+N
IF(INDEX(L1)-INDEX(L2)19,3*19
JROW=INDEX(L1)
JCOL=INDEX(L2)
NJROW=N*(JROW-1)
NJCOL=N*(JCOL-1)
DO 549 K=1,N
KJROW=K+NJROW
KJCOL=K+NJCOL
TR=AR(KJROW)
TI=AI(KJROW)
AR(KJROW)=AR(KJCOL)
AI(KJROW)=AI(KJCOL)
AR(KJCOL)=TR
AI(KJCOL)=TI
549 CONTINUE
CONTINUE
CONTINUE
CONTINUE
RETURN
END

XIRTAM
XIRTAM
If no electron density-height function is known, use the subroutine. The electron density profile is a function of the sun's zenith angle, hence a function of month, day, and hour.

FUNCTION ENNH(H)

MOD VIII OF THE D REGION

H IS HEIGHT IN KM, ENN IS ELECTRON DENSITY PER CC.

CHI IS SUNS ZENITH ANGLE IN DEGREES

USE ABSOLUTE VALUE OF CHI AS IT IS SYMMETRIC ABOUT ZERO

DIMENSION Z(80),EN(80)

COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,

1 TMER,TEMI,TMEA,TEEP, TMMR,TMMI,TMMMA,TMMP

DATA (CHIO =0.), (ENI=80.), (H1=65.), (B=.15), (H2=72.), (SCALH=3.3)

EQUIVALENCE (Z(2),CHI)

IF (CHI .EQ. CHIO) GO TO 50

CHIO =CHI

HI=65.

IF (CHI .GE. 90.3) H1=65.+(CHI-90.3)*1.03

EN1=80.

IF (CHI .GE. 90.3) EN1=0.

IF(CHI*LE.*97. AND* CHI*GT.*95.) EN1=4.*(97.-CHI)

IF(CHI*LE.*95. AND* CHI*GT.*90.) EN1=80.*10.*{(90.-CHI)/5.}

B=.15

IF (CHI .GE. 100.) B=.35

IF (CHI*GT.*70. AND* CHI*LT.*100.) B= .15+(CHI-70.)/30.*.20

IF (CHI .GE.* 90.) H2=y4.

IF (CHI .LT.* 90.) H2=y4.-22.*COS(CHI *,01745329252)

50 Q=(H-H1)/SCALH

IF (CHI*GT.* 97.) 55=52

52 CN=EN1*EXP(1.*Q-EXP(-Q))

GO TO 60

55 CN=0.

60 DN=1000.*EXP(B*(H-H2))

ENN=CN+DN

RETURN

END
Given the month, day, hour, latitude, and longitude, this subroutine calculates the sun's zenith angle which is used in the alternate Function ENN listed.

FUNCTION SOL ZEN (MONTH, DAY, TIME, LAT, LONG)
CALCULATES THE SOLAR ZENITH ANGLE AT THE GIVEN MONTH, DAY, TIME, LONG, LAT
C
C    MONTH, 1-12
C    DAY,  1-31
C    TIME, HOUR= 0-24
C    LAT=LATITUDE, RADIANS, +=NORTH, -=SOUTH
C    LONG = LONGITUDE, RADIANS EAST
C
REAL MONTH, LAT, LONG
DATA (PI=3.141592654)
GLAT=PI/180.*23.5*TRIG(3.5*(MONTH-3.)+DAY-21.)/365.*
GLON =-PI/180.*15.*TIME+PI
SOL ZEN=ACOS(SIN(LAT)*SIN(GLAT)+COS(LAT)*COS(GLAT)*COS(GLON-LONG))
RETURN
END