The Clock-Proxy Auction: A Practical Combinatorial Auction Design

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Introduction

- Many related (divisible) goods
  - Airport slots (time, airport)
  - Spectrum (bandwidth, location)
  - Electricity (duration, location, strike price)
  - Financial securities (duration)
  - Emissions (duration, type)

- A practical combinatorial auction, as an alternative to the simultaneous ascending auction (SAA)
Application: Spectrum Auction

- Trinidad and Tobago (23 June 2005)
  - Clock determines
    - Two license winners
    - Minimum price of bandwidth ($/block)
  - Proxy round determines size of licenses and specific band plan
Clock Auction

- Auctioneer names prices; bidders name only quantities
  - Price adjusted according to excess demand
  - Process repeated until market clears

- No exposure problem (package auction)
A procedure for package bidding

- Bidders input their values into “proxy agents”
- Proxy agents iteratively submit package bids, selecting best profit opportunity according to the inputted values
- Auctioneer selects provisionally-winning bids according to revenue maximization
- Process continues until the proxy agents have no new bids to submit
Clock-Proxy Auction

- A clock auction, followed by a “final round” consisting of a proxy auction
  - Bidders directly submit bids in clock auction phase
  - When clock phase concludes, bidders have a single opportunity to input proxy values
  - Proxy phase concludes the auction
Clock-Proxy Auction

- All bids are kept “live” throughout auction (no bid withdrawals)
- Bids from clock phase are also treated as package bids in the proxy phase
- All bids are treated as mutually exclusive (XOR)
- Activity rules are maintained within clock phase and between clock and proxy phases
Advantages of Clock-Proxy Auction

- **Clock phase**
  - Simple for bidders
  - Provides price discovery
    - Interdependent values
    - Economize on package evaluation costs

- **Proxy phase**
  - Efficient allocations
  - Competitive revenues
  - Reduces opportunities for collusion
Clock Auction
Practical implementation of the fictitious “Walrasian auctioneer”

- Auctioneer announces a price vector
- Bidders respond by reporting quantity vectors
- Price is adjusted according to excess demand
- Process is repeated until the market clears
Simultaneous Clock Auction

- **Strengths**
  - Simple for bidders
  - Provides highly-usable price discovery
  - Yields similar outcome as SAA, but faster and fewer collusive opportunities
  - A package auction without complexity

- **Weaknesses**
  - Limits prices to being linear
  - Therefore should not yield efficient outcomes
Recent Clock Auctions

- EDF generation capacity (virtual power plants)
  - 16 quarterly auctions (Sep 2001 – present)
- Electrabel generation (virtual power plants)
  - 7 quarterly auctions (Dec 2003 – present)
- Ruhrgas gas release program
  - 3 annual auctions (2003 – present)
- Trinidad and Tobago spectrum auction
  - 1 auction (June 2005)
- Federal Aviation Administration airport slot auction
  - 1 demonstration auction (Feb 2005)
- UK emissions trading scheme
  - World’s first greenhouse gas auction (Mar 2002)
- GDF and Total gas release program
  - 2 auctions (Oct 2004)
Recent Clock Auctions

- New Jersey basic generation service
  - 5 annual auctions (2002 – present)

- Texas electricity capacity
  - 16 quarterly auctions (Sep 2001 – present)

- Austrian gas release program
  - 3 annual auctions (2003 – present)

- Nuon generation capacity
  - 1 auction (September 2004)
EDF Generation Capacity Auction

EDF
Electricité de France

MDI
market design inc.
Typical EDF Auction

- Number of products
  - Two to four groups (baseload, peakload, etc.)
  - 20 products (various durations)

- Number of bidders
  - 30 bidders
  - 15 winners

- Duration
  - Eight to ten rounds (one day)

- €300 million in value transacted in auction
Electrabel VPP Capacity Auction
Typical Electrabel Auction

- **Number of products**
  - Two groups (baseload, peakload)
  - 20 products (various durations and start dates)

- **Number of bidders**
  - 14 bidders
  - 7 winners

- **Duration**
  - Seven rounds (*one day*)

- **€100 million in value transacted in auction**
Issue 1: Discrete bidding rounds are helpful for maintaining legally-binding bids, but they can yield slow auctions or “overshoot”

**SOLUTION:** Intra-round bids: If the (end) price of Round 3 is €19,000 and the (end) price of Round 4 is €19,500 for baseload, and if the (end) price of Round 3 is €10,300 and the (end) price of Round 4 is €10,600 for peakload, then bidders in Round 4 submit demand curves for all price pairs from (€19,000 , €10,300) to (€19,500 , €10,600).
1 Product – Dealing with Discreteness

Closing Price:
- P6
- P5
- P4
- P3
- P2
- P1

Overshoot
- Round 6
- Round 5
- Round 4
- Round 3
- Round 2
- Round 1

Price

Supply

Aggregate Demand

MW
1 Product introducing intra-round bidding
1 product – Individual bids with intra-round bidding
1 product – Aggregate demand with intra-round bidding
Sample (redacted) data 1
Issue 2: Treatment of bids which would make aggregate demand < supply

- Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce its demand
  - Naive approach: Prevent the reduction

- Example: For a particular item, demand > supply, but demand < supply at next increment
  - Naive approach: Ration the bidders
Issue 2: Treatment of bids which would make aggregate demand < supply

- Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce its demand
  - Difficulty: Creates an exposure problem

- Example: For a particular item, demand > supply, but demand < supply at next increment
  - Difficulty: Creates an exposure problem
Issue 2: Treatment of bids which would make aggregate demand < supply

- **Example:** For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce its demand
  - Our approach: Allow the reduction

- **Example:** For a particular item, demand > supply, but demand < supply at next increment
  - Our approach: No rationing
Issue 2: Treatment of bids which would make aggregate demand < supply

- Bids in clock phase are treated as package bids
- Thus, our clock auctions are, in fact, combinatorial auctions

- Advantage: No exposure problem

- Disadvantage: Potential significant undersell
  (*But not a problem in the clock-proxy auction, since clock phase followed by a final proxy round*)
Issue 3: Activity rules

- Prevent a bidder from hiding as a “snake in the grass” to conceal its true interests

- Standard approaches:
  - No activity rule (laboratory experiments)
  - Monotonicity in quantities (SAA and clock auctions in practice)
Issues in Implementing Clock Auctions

Issue 3: Activity rules

- Revealed-preference activity rule (advocated here)

- Compare times $s$ and $t$ ($s < t$),
  Prices: $p^s, p^t$   Demands: $x^s, x^t$

  - At time $s$, $x^s$ is better than $x^t$: $v(x^s) - p^s \cdot x^s \geq v(x^t) - p^s \cdot x^t$
  - At time $t$, $x^t$ is better than $x^s$: $v(x^t) - p^t \cdot x^t \geq v(x^s) - p^t \cdot x^s$
  - Adding inequalities yields the RP activity rule:

    \[(RP) \quad (p^t - p^s) \cdot (x^t - x^s) \leq 0.\]
Issue 3: Activity rules

- Revealed-preference activity rule (advocated here)

- Bid placed at time $t$ must satisfy (RP) with respect to its prior bids at all prior times $s$ ($s < t$):

$$\text{(RP)} \quad (p^t - p^s) \cdot (x^t - x^s) \leq 0.$$ 

- One can also apply a “relaxed” RP in proxy phase (with respect to bids in the clock phase)
Proxy Auction
Package Bidding

- Package bidding often motivated by complements
- Even without complements, package bidding may improve outcome by eliminating “demand reduction”
  - In SAA, bidders may have strong incentives to reduce demands in order to end auction at low prices
Ascending Proxy Auction

- Each bidder reports its values (and constraints) to a "proxy agent", in a sealed-bid round
- The proxy agents bid in an auction in "virtual time"
- The proxy agent’s rule: submit the allowable bid that, if accepted, would maximize the bidder’s payoff (evaluated according to its reported values)
- The virtual auction ends after a round with no new bids by the proxy agents
The coalitional form game is \((L, w)\), where...

- \(L\) denotes the set of players.
  - the seller is \(l = 0\)
  - the other players are the bidders

- \(w(S)\) denotes the value of coalition \(S\):
  - If \(S\) excludes the seller, let \(w(S)=0\)
  - If \(S\) includes the seller, let
    \[
    w(S) = \max_{x \in X} \sum_{i \in S} v_i(x_i)
    \]

- The \(Core(L, w)\) is the set of all profit allocations that are \textit{feasible} for the coalition of the whole and \textit{cannot be blocked} by any coalition \(S\)
Outcomes in the Core

**Theorem:** The payoff vector resulting from the proxy auction is in the core relative to the reported preferences.

**Interpretations:**

- Core outcome assures competitive revenues for seller
- Core outcome assures allocative efficiency (ascending proxy auction is not subject to inefficient demand reduction)
**Theorem**: If \( \pi \) is a bidder-Pareto-optimal point in \( \text{Core}(L, w) \), then there exists a full information Nash equilibrium of the proxy auction with associated payoff vector \( \pi \).

These equilibria may be obtained using strategies of the form: bid your true value minus a nonnegative constant on every package.
Case of Substitutes

- If goods are substitutes, then Vickrey payoff profile is unique bidder-Pareto-optimal point in core.
- Outcome of the ascending proxy auction coincides with outcome of the Vickrey auction.

*Diagram*

- Core Payoffs for 1 and 2
- Bidder #1 Payoff
- Bidder #2 Payoff
- Vickrey Payoff Vector

Mathematical expressions:
- \( v_1 + v_2 \leq w(L) - w(L \setminus 12) \)
- \( w(L) - w(L \setminus 1) \)
Case of Non-Substitutes

- If goods are not substitutes, then Vickrey payoff profile is not in core

- Ascending proxy auction yields a different outcome from the Vickrey auction (one with higher revenues)
Proxy Auction Avoids Vickrey Problems

- In Vickrey auction:
  - Adding a bidder can reduce revenues
  - Using a shill bidder can be profitable
  - Losing bidders can profitably collude

- Proxy auction avoids these problems
Clock-Proxy Auction
Clock-Proxy Auction

- A simultaneous clock auction is conducted, with a revealed-preference activity rule imposed on bidders, until (approximate) clearing is attained.

- A proxy auction is conducted as a “final round”
  - Bids submitted by proxy agents are restricted to satisfy a relaxed revealed-preference activity rule based on competitive conditions.
  - Bids from clock phase are also treated as “live” package bids in proxy phase.
  - All package bids (clock and proxy) are treated as mutually exclusive, and auctioneer selects as provisionally-winning the bids that maximize revenues.
Relaxed Revealed Preference Activity Rule

- Let \( s \) be a time in clock phase and \( t \) a time in proxy phase
- Package \( S \) is bid on at time \( s \) and \( T \) is bid on at time \( t \)
- \( P^s(S) \) and \( P^s(T) \) package prices of \( S \) and \( T \) at time \( s \)
- \( P^t(S) \) and \( P^t(T) \) package prices of \( S \) and \( T \) at time \( t \)
- At every time \( t \) in the proxy phase, the bidder can bid on the package \( T \) only if (RRP) is satisfied for every package \( S \) bid at time \( s \) in the clock phase

(\text{RRP}) \quad \alpha [P^t(S) - P^s(S)] \geq P^t(T) - P^s(T)

- \( \alpha > 1 \) is parameter (closer to 1 if more competitive environment)
- For \( \alpha = 1 \), price of \( S \) increased more than price of \( T \); otherwise \( S \) would be more profitable than \( T \).

- Alternatively, state RRP as a constraint on valuations reported to proxy:

\[ v(T) - P^s(T) \leq \alpha \left( v(S) - P^s(S) \right) \]
Why Not Use the Proxy Auction Only?

- Clock auction phase yields price discovery
- Feedback of linear prices is extremely useful to bidders
- Clock phase makes bidding in the proxy phase vastly simpler
  - Focus decision on what is relevant
  - See what you don't need to consider
  - See what looks like good possibilities
Why Not Use the Clock Auction Only?

- Proxy auction ends with core outcome
  - Efficient allocation
  - Competitive revenues
- No demand reduction
- Collusion is limited
  - Relaxed activity rule means allocation still up for grabs in proxy phase
Advantages of the Clock over the SAA

- Clock auction is a fast and simple process (compared to the simultaneous ascending auction)
  - Only provide information relevant for price and quantity discovery (excess demand)
  - Takes advantage of substitutes (one clock for substitute licenses)
  - Example:
    - proposed 90 MHz of 3G spectrum in 5 blocks: 30, 20, 20, 10, 10
    - clock alternative: 9 or 18 equivalent blocks per region
  - Fewer rounds
    - Get increment increase for all items, rather than having to cycle through over many rounds
    - “Intra-round bids” allow larger increments, but still permit expression of demands along line segment from start-of-round price to end-of-round price
Advantages of the Clock over the SAA

- Clock auction limits collusion (compared to the simultaneous ascending auction)
  - Signaling how to split up the licenses greatly limited
    - No retaliation (since no bidder-specific information)
    - No stopping when obvious split is reached (since no bidder specific information)
  - Fewer rounds to coordinate on a split
Advantages of the Clock Phase

- No exposure problem (unlike SAA)
  - As long as at least one price increases, bidder can drop quantity on other items
  - Bidder can safely bid for synergistic gains
  - Bid is binding only as full package

- Limited threshold problem (unlike ascending package auction)
  - Clocks controlled by auctioneer: no jump bids; large bidder cannot get ahead
  - Linear pricing: small bidders just need to meet price on single item
Clock-Proxy Auction

- Combines advantages of
  - Clock auction
  - Proxy auction

- Excellent price discovery in clock phase simplifies bidder decision problem

- Proxy phase enables bidders to fine-tune allocation based on good price information
Advantages of Clock-Proxy Auction

- **Clock**
  - Take linear prices as far as they will go
  - Simplicity and flexibility for bidders and auctioneer
  - Expand substitution possibilities
  - Minimize scope for collusion
  - No exposure problem; no threshold problem

- **Proxy**
  - Core outcome
    - Efficiency
    - Substantial seller revenues