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A Wave Hop Propagation Program for an Anistropic Ionosphere

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J. E. HERMAN

BOULDER,
COLORADO
APRIL 1971

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TABLE OF CONTENTS

FOREWORD

This work was performed for the Defense Atomic Support Agency, under the direction of Mr. Dow E. Evelyn, HQ, DASA.

A WAVE HOP PROPAGATION PROGRAM FOR AN ANISOTROPIC IONOSPHERE

Leslie A. Berry and J. E. Herman

This report documents a digital computer program for computing the propagation of LF and VLF radio waves using the wave hop theory. It supersedes NBS Report 8889. The documentation includes a description of the physical model, detailed mathematical formulas, a main function flow chart, a listing of the FORTRAN source deck, and the input and output of a sample case.

1. INTRODUCTION

This report documents a computer program for the full wave calculation of LF and VLF radio propagation under and anisotropic ionosphere using the wave hop theory. It supersedes NBS Report 8889, "FORTRAN Programs for Full-Wave Calculation of LF and VLF Radio Propagation" (Berry and Chrisman, 1965a). The revision is necessary because the original program was based on a formulation that is incorrect for an anisotropic ionosphere (see Berry, et al. (1969) for details). The error was significant if the magnitudes of the ionospheric polarization coupling (conversion) coefficients were comparable to those of the reflection coefficients. The program and all its subroutines have been completely rewritten in a more modern version of FORTRAN (CDC FORTRAN 63) and a number of improvements suggested by users have been incorporated.

The next section explains the concepts and model used, so that you will know when it is appropriate to use the program, how to determine the model, and what input information you must provide. Section 3 contains detailed mathematical formulas used, and section 4 describes the computer implementation. This last section includes a test case and a listing of the FORTRAN source deck.

An ionospheric reflection coefficient program which can be used to generate input data for the propagation program is described in the appendix.

2. THE PHYSICAL PROBLEM AND THE MATHEMATICAL MODEL

The problem solved by the computer program can be stated as follows:

> Given: A ground-based source of vertically polarized radio waves[1] of known frequency in the LF-VLF band, the electrical constants of the ground and the reflection characteristics of the ionosphere along a great circle path.
>
> Calculate: The amplitude and phase of the propagated vertical electric field on the earth's surface as a function of distance along the path.

The geometry of the assumed model is shown in figure 1. The earth is a smooth sphere with radius a and electrical constants σ (conductivity) and ϵ (dielectric constant). The ionosphere is concentric with known reflection properties characterized by the ionospheric reflection coefficient matrix $\underline{T}(\varphi)$, where φ is the angle of incidence of a wave on the ionosphere. The height of the ionosphere above the earth, h, can be thought of as the virtual, or phase, height. Distance, d, is measured along the ground; we also use the angular distance $\theta = d/a$.

We compute the vertical (radial) electric field at O due to a vertically polarized source at S. The field is the vector sum of several components which travel to O via different paths:

$$E(d) = \sum_{j=0}^{J} E_j(d) \; , \tag{1}$$

where the subscript j counts the number of times the energy has been reflected from the ionosphere. Thus, E_0 is the ground wave that travels directly along the surface of the earth from S to O. E_1 is the "first hop" (shown as the solid line in figure 1) which has reflected once from the ionosphere; E_2 is the "second hop" (shown as the dashed line in

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[1] Lewis (1970) derives wave hop formulas for an elevated, arbitrarily-oriented dipole.

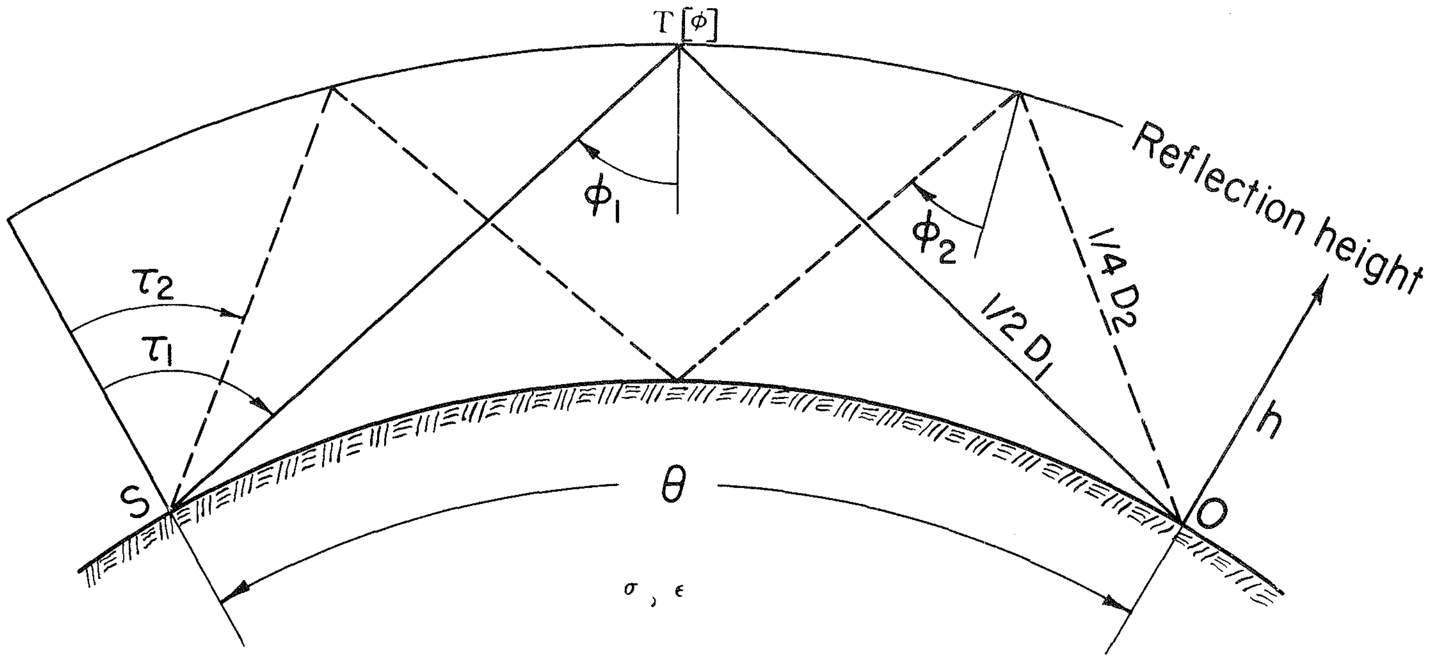

Figure 1. Geometry for wave hop propagation model.

figure 1); and so forth. The angle of incidence of the path of the j-th hop on the ground is denoted τ_j; the angle of incidence on the ionosphere is denoted φ_j.

The theory is developed by Wait (1961), Berry (1964), and Berry et al. (1969), and the propagation characteristics of the wave hops are discussed by Berry (1964, 1967) and Johler (1970). Wave hop theory is most appropriate at LF (30-300 kHz) for distances less than, say, 6000 km. At lower frequencies a VLF waveguide mode theory (Wait, 1968 and references cited therein; Pappert, 1968) program is usually more appropriate, especially for long paths, although the wave hop program will compute fields correctly down to a few kilohertz and out to distances beyond 10,000 km. This conclusion is illustrated by Morfitt and Halley (1970) who compare fields calculated using this wave hop theory with fields calculated using the NEL mode theory program (Sheddy et al., 1968).

To use the program, you must specify:

(a) The transmitter's operating frequency and effective radiated power.

(b) The ground conductivity and relative dielectric constant. These are assumed to be constant along the path. If the conductivity varies significantly along the path, the geometric mean of the path conductivities is a useful approximation. Morgan (1968) provides a worldwide VLF effective-conductivity map which is also useful at low LF. At high LF and above, conductivity maps prepared for MF frequencies, such as that prepared by Barghausen, et al. (1966) may be more appropriate.

(c) Ionospheric reflection coefficients as a function of angle of incidence. The coefficients should be given for the entire range of angles of incidence that will be encountered by the program during execution. Belrose (1968), Watt (1967), and Berry and Chrisman (1965b) show graphs of the angle of incidence for various distances and reflection heights. The appendix of this report contains a reflection coefficient program that can be used to compute reflection coefficients for any given ionospheric profile, or reflection coefficients from different sources can be used. Belrose (1968) shows empirically derived reflection coefficients for frequencies in the upper LF and MF bands.

(d) The distances of interest. These are given by a minimum distance, a distance increment, and a maximum distance.

(e) The number of hops to be computed and summed. The number may be different for each of five (or fewer) distance ranges, because fewer hops are necessary at short ranges. The number of hops necessary for the series in (1) to converge depends on the amplitude of the ionospheric reflection coefficients, the ground conductivity, and the frequency. The amplitude of the individual terms in (1) are printed out, so it is possible to determine, after the fact, if enough terms were used, but advance determination of the minimum number of hops necessary still requires judgement. For daytime ionospheric reflection coefficients a crude rule-of-thumb is: number of hops = (distance in Mm) + 1. Fewer hops are needed at high LF, more are needed at night.

3. DETAILS OF THE MATHEMATICAL SOLUTION

3.1 Notation

f = frequency, Hz

P_r = effective radiated power, watts

d = distance along the great circle path, m

a_1 = earth's radius $\approx 6.36739\ (10^6)$, m

h = phase reference height of ionosphere (loosely, "reflection height")

c = speed of light $\approx 2.997925\ (10^8)$ m/s

$\mu_0 = 4\pi\ (10^{-7})$

σ = ground conductivity, mho/m

ϵ = relative dielectric constant of ground

T_{ee}, T_{em}, T_{me}, T_{mm} are ionospheric reflection and conversion coefficients (Johler and Harper, 1962). The subscripts indicate which vector (electric or magnetic) is in the plane of incidence; the first subscript refers to the incident wave and the second to the reflected wave. The equivalence to a more common notation is: $T_{ee} \equiv {}_{\parallel}R_{\parallel}$, $T_{em} \equiv {}_{\parallel}R_{\perp}$, $T_{me} \equiv {}_{\perp}R_{\parallel}$, and $T_{mm} \equiv {}_{\perp}R_{\perp}$.

$$\varphi = \text{the angle of incidence on the ionosphere} \tag{2}$$

$$\omega = 2\pi f,\ k = \omega/c \tag{3}$$

$$k_2 = \sqrt{k^2\epsilon - i\mu_0\omega c} = k\sqrt{\epsilon - i\,\frac{\mu_0 c^2\sigma}{\omega}} \tag{4}$$

$$a_2 = a_1 + h \tag{5}$$

$$v = (ka_1/2)^{\frac{1}{3}} \tag{6}$$

$$x = v\theta\ ,\ y = (kh/v) \tag{7}$$

$$z = 1.25/v^2$$

$$q_e = -iv\frac{k}{k_2}\sqrt{1-\left(\frac{k}{k_2}\right)^2}$$

$$q_m = -iv\frac{k_2}{k}\sqrt{1-\left(\frac{k}{k_2}\right)^2} \tag{8}$$

$$\theta = d/a_1 \text{ , } x = v\theta \tag{9}$$

$$F = 30\sqrt{\frac{\pi P_r}{120}}\text{ , } \quad G = \frac{e^{-ikd+i\pi/4}}{d}\sqrt{\frac{\theta x}{\pi \sin\theta}}\cdot\frac{F}{2}\text{ .} \tag{10}$$

3.2 The Ground-Wave

The first term of (1) is the ground wave, E_0. It is given by (compare Wait (1962), Fock (1964))

$$E_0(d) = -4\pi G\sum_s (1+zt)\left(1+\frac{3+i\cot\theta}{8ka_1+vt_s}\right)\frac{e^{-ixt_s}}{t_s-q_e^2}\text{ .} \tag{11}$$

The t_s satisfy

$$W_1'(t_s) - q_e W_1(t_s) = 0\text{ ,} \tag{12}$$

where $W_1(t)$ is the Airy function of the first kind defined by Wait (1962), and the t_s are numbered in order of increasing magnitude of the imaginary part. The first two factors in the s-th term differ from unity by a few percent, at most, and are frequently neglected in ground wave calculations.

3.3 The Wave Hops

For $j \geq 1$ in (1), the wave hops are (Berry, et al., 1969)

$$E_j = I_{j1}T_{ee}^j + \sum_{M=2}^{j} I_{jM}C_{jM}\text{ ,} \tag{13}$$

where

$$C_{jM} = \sum_{k=1}^{M-1} a_{jMk} \, T_{ee}^{j+1-M-k} \, (T_{em} \, T_{me})^{k} \, T_{mm}^{M-1-k} \, . \tag{14}$$

Then

$$a_{jM1} = j - M + 1 \, , \tag{15}$$

and

$$a_{jMk} = \frac{(j+2-M-k)\,(M-k)}{k(k-1)} \, a_{jM(k-1)} \quad \text{for } k \geq 2 \, .$$

Both I_{jM} and the T_{ik} are implicit functions of d. The functions are made explicit below, beginning with the path integrals, I_{jM}.

The basic formula is (Berry, et al., 1969)

$$I_{jM} = G \int_{\Gamma} (1+zt) \, e^{-ixt} \, W_1(t) \, W_2(t) \, (1+R_e)^2 \, p^j \, R_e^{j-M} \, R_m^{M-1} \, dt \, , \tag{16}$$

where the ground reflection coefficients are

$$R_i = - \frac{W_2'(t)/W_2(t) - q_i}{W_1'(t)/W_1(t) - q_i} \, , \quad i = e \text{ or } m \, , \tag{17}$$

and

$$p = \frac{W_2(t) \, W_1(t-y)}{W_1(t) \, W_2(t-y)} \quad . \tag{18}$$

Γ runs from ∞ to 0 and down into the third quadrant to $\infty \, e^{-i2\pi/3}$. However, (16) is an approximation to an integral involving Hankel functions of complex order (Berry, 1964; Berry and Chrisman, 1965c), and the approximation is valid only if $\cos \tau_j$ is small (refer to figure 1).

3.3.1 Geometrical Optics

For short distances, $\cos \tau_j$ is not small so we use the geometrical optics formula:

$$I_{jM} \approx -i\, F \frac{e^{-ikD_j}}{D_j}\; B_j \sin^2 \tau_j (1+\hat{R}_e)^2\, \hat{R}_e^{j-M}\, \hat{R}_m^{M-1} \;, \qquad (19)$$

where

$$D_j = 2j \sqrt{2a_1a_2(1 - \cos \frac{\theta}{2j}) + h^2} \;, \qquad (20)$$

$$\sin \tau_j = \frac{2j}{D_j}\, a_2 \sin \frac{\theta}{2j} \;, \qquad (21)$$

$$\cos \varphi_j = \frac{D_j + 2ja_1 \cos \tau_j}{2ja_2} \;, \qquad (22)$$

and the convergence-divergence coefficient is

$$B_j = \frac{a_2}{a_1} \sqrt{\frac{D_j \sin \tau_j \cos \varphi_j}{a_2 \sin \theta \cos \tau_j}} \;. \qquad (23)$$

The Fresnel ground reflection coefficients are

$$\hat{R}_i = \frac{s + q_i}{s - q_i} \;, \text{ where } s = iv \cos \tau_j \qquad (24)$$

3.3.2 Saddle Point Approximation.

The relationship between (19) and (16) was shown by Wait (1961). Following his derivation, change the contour Γ so that we integrate from $-\infty$ to ∞. Since we cross no singularities with this move, it only changes the sign of (16). Then, if $(-t) >> 1$,

$$W_k(t) \approx (-\bar{t})^{\frac{1}{4}} \exp \left[(-1)^k i\left(\frac{2}{3}(-t)^{\frac{3}{2}} + \frac{\pi}{4}\right)\right]$$

and

$$W_k' \approx (-1)^{k-1}\, i(-t)^{\frac{1}{2}}\, W_k(t) \;.$$

Substitution into (16) yields

$$I_{jM} \approx -G \int_{-\infty}^{\infty} e^{-i\Omega(t)}(1+zt)\,(-t)^{\frac{1}{2}}\,(1+R_e)^2\, R_e^{j-M}\, R_m^{M-1}\, dt\,, \quad (26)$$

where

$$\Omega(t) = xt - j\,\frac{4}{3}\,(-t)^{\frac{3}{2}} + \frac{4}{3}\, j(y-t)^{\frac{3}{2}}\,, \quad (27)$$

and now

$$R_i(t) \approx \frac{(-t)^{\frac{1}{2}} - iq_i}{(-t)^{\frac{1}{2}} + iq_i}\,.$$

The saddle point approximation to (26) is

$$I_{jM} \approx -i\,F\,\frac{e^{-ikd}}{d}\,\sqrt{\frac{\theta}{\sin\,\theta}}\,(1-z\alpha^2)(1+\frac{x}{2j\alpha})\,(1+R_e)^2\, R_e^{j-M}\, R_m^{M-1}, \quad (28)$$

where

$$\alpha = (-t_0)^{\frac{1}{2}} = \frac{4j\,y - x^2}{4j\,x}\,. \quad (29)$$

It can be shown (Wait, 1961) that, for $\cos\,\tau_j << 1$,

$$(-t_0)^{\frac{1}{2}} \approx v\,\cos\,\tau_j\,,$$

$$\Omega(t_0) \approx k(D_j - d)\,,$$

$$(1 + \frac{x}{2j\alpha})^{\frac{1}{2}} \approx B_j\,,$$

$$(1 - z\alpha^2) \approx \sin^2\,\tau_j\,,$$

and

$$d\,\frac{\sin\,\theta}{\theta} \approx D_j\,,$$

so (19) and (28), and hence (16), are approximately equal for the conditions $(-t_0) >> 1$ and $\cos \tau_j << 1$. This small region is large enough to overlap (16) and (19).

3. 3. 3 Numerical Integration.

When τ_j approaches $\pi/2$, (16) must be integrated numerically. We use the Wronskian (Wait, 1962)

$$W_1'(t)\, W_2(t) - W_2'(t)\, W_1(t) = 2i$$

and (17) to derive

$$(1 + R_e)^2 = \frac{-4}{W_2^2(t)\,(W_1'(t) - q_e\, W_1(t))^2}\ , \tag{30}$$

and hence

$$I_{jM} = G \int_{\Gamma} (4)(-1)^j (1+zt)\, e^{-ixt} \left(\frac{W_1(t-y)}{W_2(t-y)}\right)^j \frac{(E_2(t,q_e))^{j-M}}{(E_1(t,q_e))^{j-M+2}} \left(\frac{E_2(t,q_m)}{E_1(t,q_m)}\right)^{M-1} dt\ , \tag{31}$$

where

$$E_k(t, q_i) = W_k'(t) - q_i\, W_k(t)\ . \tag{32}$$

3. 3. 4 Residue Series.

The integrand in (31) has poles of order j-M+2 wherever

$$E_1(t,\ q_e) = 0\ ,$$

and poles of order M-1 wherever

$$E_1(t,\ q_m) = 0\ .$$

Zeroes for both functions exist inside the contour Γ; they are the well-known solutions used for calculation of ground wave propagation over a spherical earth or diffraction by a finitely conducting sphere (Wait, 1962; Fock, 1965). The zeroes of $W_2(t-y)$ are all in the upper half plane outside the contour Γ. Thus, I_{jM} can be evaluated by summing a residue series:

$$I_{JM} = 2\pi i\, G \sum_{s} \mathrm{Res}_{JM}(t_s)\ . \tag{33}$$

We will first discuss methods for calculating residues of high order poles; then return to the problem of finding the pole locations t_s.

Assume that we have an integral

$$I = \oint \frac{A(t)}{B(t)}\, dt\ , \tag{34}$$

where A(t) is analytic on and inside the contour and B(t) has a zero of order N at $t = t_0$ inside the contour. Expand A(t), B(t), and their ratio in Laurent series around t_0 and equate coefficients of like powers to obtain (see for example Kaplan (1952), pp. 564-565)

$$\begin{array}{ll} b_0\, \hat{a}_0 & = a_0 \\ b_1\, \hat{a}_0 + b_0\, \hat{a}_1 & = a_1 \\ b_2\, \hat{a}_0 + b_1\, \hat{a}_1 + b_0\, \hat{a}_2 & = a_2 \\ \quad\cdot \qquad \cdot \qquad \cdot & \quad\cdot\ , \\ \quad\cdot \qquad \cdot \qquad \cdot & \quad\cdot \\ b_{N-1}\, \hat{a}_0 + b_{N-2}\, \hat{a}_1 + \ .\ .\ .\ .\ .\ .\ .\ + b_0\, \hat{a}_{N-1} & = a_{N-1} \end{array} \tag{35}$$

where

$$a_i = \frac{\left.\dfrac{d^i}{dt^i}\,(A(t))\right|_{t=t_0}}{i!}\ , \tag{36}$$

and

$$b_i = \frac{\left.\dfrac{d^{i+N}}{dt^{i+N}}\,(B(t))\right|_{t=t_0}}{(i+N)\ !}\ ,$$

are the coefficients of the Laurent series for A(t) and B(t) respectively, and the $\hat{a}$ are the coefficients of the series for A/B. By definition

$$\mathrm{Residue}\ (t_0) = \hat{a}_{N-1}. \tag{37}$$

The set of equations (35) can be solved recursively:

$$\hat{a}_0 = a_0/b_0; \text{ and } \hat{a}_i = (a_i - \sum_{\ell=0}^{i=0} b_{i-\ell}\, \hat{a}_\ell)/b_0 . \tag{38}$$

Comparing (34) and (31), we see that if

$$E_1(t, q_e) = 0 \text{ ("}q_e\text{ poles") },$$

$$A(t) = 4(-1)^j\,(1 + zt)\, e^{-ixt} \left(\frac{W_1(t-y)}{W_2(t-y)}\right)^j \frac{E_2^{j-M}(t, q_e)\, E_2^{M-1}(t, q_m)}{E_1^{M-1}(t,\, q_m)}, \tag{39}$$

and

$$B(t) = (E_1(t, q_e))^{j-M+2} .$$

If $E_1(t, q_m) = 0$, ("q_m poles"), then $N = M-1$ (there are no q_m poles for $M = 1$),

$$A(t) = 4(-1)^j(1 + zt)\, e^{-ixt} \left(\frac{W_1(t-y)}{W_2(t-y)}\right)^j \frac{E_2^{j-M}(t, q_e)\, E_2^{M-1}(t, q_m)}{E_1^{j-M+2}(t, q_e)}, \tag{40}$$

and

$$B(t) = (E_1(t, q_m))^{M-1} .$$

Taking high order derivatives of products of several functions (such as A(t)) analytically is very tedious, but such derivatives can easily be evaluated numerically on a computer by repeated application of Liebnitz's rule:

$$(gh)^{(k)} = \sum_{m=0}^{k} \frac{k!}{m!\,(k-m)}\, g^{(m)}\, h^{(k-m)} , \tag{41}$$

where g and h are functions of t and

$$g^{(m)} = \frac{d^m}{dt^m}\, g .$$

In order to apply (41) to (39) and (40), we must be able to compute all derivatives of $(E_1(t, q))^{-1}$. In (41), let $h = g^{-1}$ so that $gh = 1$, and $(gh)^{(k)} = 0$, if $k > 0$. Then (41) becomes

$$0 = g^{(0)} h^{(k)} + \sum_{m=1}^{k} \frac{k!}{m!\,(k-m)!} g^{(m)} h^{(k-m)} .$$

Notice that $g^{(0)} = g = 1/h$. Solve for $h^{(k)}$:

$$h^{(k)} = (g^{-1})^{(k)} = -h \sum_{m=1}^{k} \frac{k!}{m!\,(k-m)!} g^{(m)} h^{(k-m)} , \tag{42}$$

which is a recursive formula for derivatives of g^{-1} in terms of derivatives of g.

Similarly, recursive formulas for the b_i for a pole of order N are:

$$b_0 = (E_1^{(1)}(t_0, q_p))^N ,$$

where p = e or m, as appropriate, and

$$b_i = (E_1^{(1)}(t_0, q_p))^{-1} \sum_{k=1}^{i} \frac{k(N+1) - i}{i(k+1)!} b_{i-k} E_1^{(k+1)}(t_0, q_p) . \tag{43}$$

Finally, all derivatives of the functions involved in (39) and (40) can be found recursively. Assume that $W_k(t)$ and $W_k'(t)$ are given. Then the differential equation for W_k $[W_k^{(2)}(t) = t W_k(t)]$ and straightforward mathematical induction show that

$$W_k^{(n)}(t) = t W_k^{(n-2)} + (n-2) W_k^{(n-3)}(t) , \text{ for } n \geq 2 . \tag{44}$$

Using (32),

$$E_k^{(n)}(t, q) = W_k^{(n+1)}(t) - qW_k^{(n)}(t) . \tag{45}$$

Of course,

$$(e^{-ixt})^{(n)} = (-ix)^n \, e^{-ixt} \, . \tag{46}$$

We now return to the problem of finding the location of the poles, that is, the zeroes of $E_1(t, q)$. We use Newton's iteration:

$$t \approx \hat{t} - \frac{E_1(\hat{t}, q)}{E_1'(\hat{t}, q)} = \hat{t} - \frac{W_1'(\hat{t}) - q\, W_1(\hat{t})}{\hat{t}\, W_1(\hat{t}) - q\, W_1'(\hat{t})} \quad , \tag{47}$$

where $\hat{t}$ is an approximation to t. For first approximations, we use the known zeroes of $W_1'(t)$, if q is small, and the zeroes of $W_1(t)$ if q is large. The first ten such zeroes are listed in Subroutine TW, which also contains approximate formulas for all the other zeroes.

3.4 Ionospheric Reflection Coefficients

The ionospheric reflection coefficients, T, are functions of the angle of incidence, φ, which depends on distance and the hop number. When the geometrical-optics formula (19) is used, φ is computed with (22). When (28) or (31) is used, φ is given by (29), $\cos \tau_j = (-t_0)^{\frac{1}{2}}/v$, and $\sin \varphi = a_1/a_2 \sin \tau_j$. When I_{jM} is evaluated with the residue series (33), we use these same formulas except now t_0 is the first q_e pole, and hence is complex. Continuation of the ionospheric reflection coefficients into the complex φ-plane is accomplished by approximating the input values along the real axis with analytic functions and continuing these functions analytically.

Following Wait (1962), we approximate T_{ee} and T_{mm} by an exponential

$$T \approx - \exp(A \cos \varphi) \, , \tag{48}$$

where A is a complex number determined from the input data:

$$\text{Real}\,(A) = \log\,(|T|)/\cos \varphi \, , \tag{49}$$

$$\text{Imag}\,(A) = (\text{Phase}\,(T) - \pi)/\cos \varphi \, .$$

The polarization coupling coefficients T_{em} and T_{me} are approximated by fitting their amplitude and phase with linear functions.

4. NOTES ON COMPUTER IMPLEMENTATION

4.1 Program Organization and Flow

Figure 2 shows the large-block program organization and flow. The numbers in the blocks are the statement numbers where the block function begins. Details of program organization and flow are best determined from the statements and comments in the program listing in section 4.4.

4.1.1 Numerical Integration Branch.

We use 48-point Gaussian quadrature to integrate (31) on a finite portion of the contour Γ. First, for $t >> 1$,

$$W_{\kappa}(t) \approx t^{-\frac{1}{4}} \exp(\frac{2}{3} t^{\frac{3}{2}}) \ , \tag{50}$$

and

$$W_{\kappa}'(t) \approx t^{\frac{1}{2}} W_{\kappa}(t) \ ,$$

so the magnitude of the integrand in (31) is about $4t^{\frac{1}{2}} \exp(-\frac{4}{3} t^{\frac{3}{2}})$. We consider the contribution beyond $t = 4$ to be negligible, so the first portion of the contour of integration runs from 4 to 0 along the real axis. The second portion of the contour is a straight line from 0 to $K(-4-i)$, where K is chosen so that the real part of $-ixt$ is -9 for the shortest distance for which numerical integration would be used. Specifically, $K = -9/(2\sqrt{4+y} - 4)$. The slope of this contour was chosen empirically to be optimum for convergence of the integrand. The rest of the infinite contour Γ contributes little to the integral, so we ignore it.

4.1.2 Residue Series and the Subroutine LPAINR.

The residues are calculated in the subroutine LPAINR. Since only the factor e^{-ixt} in $A(t)$ (see (39) and (40)) depends on distance, all necessary derivatives of $A(t)/e^{-ixt}$ are computed and stored on the

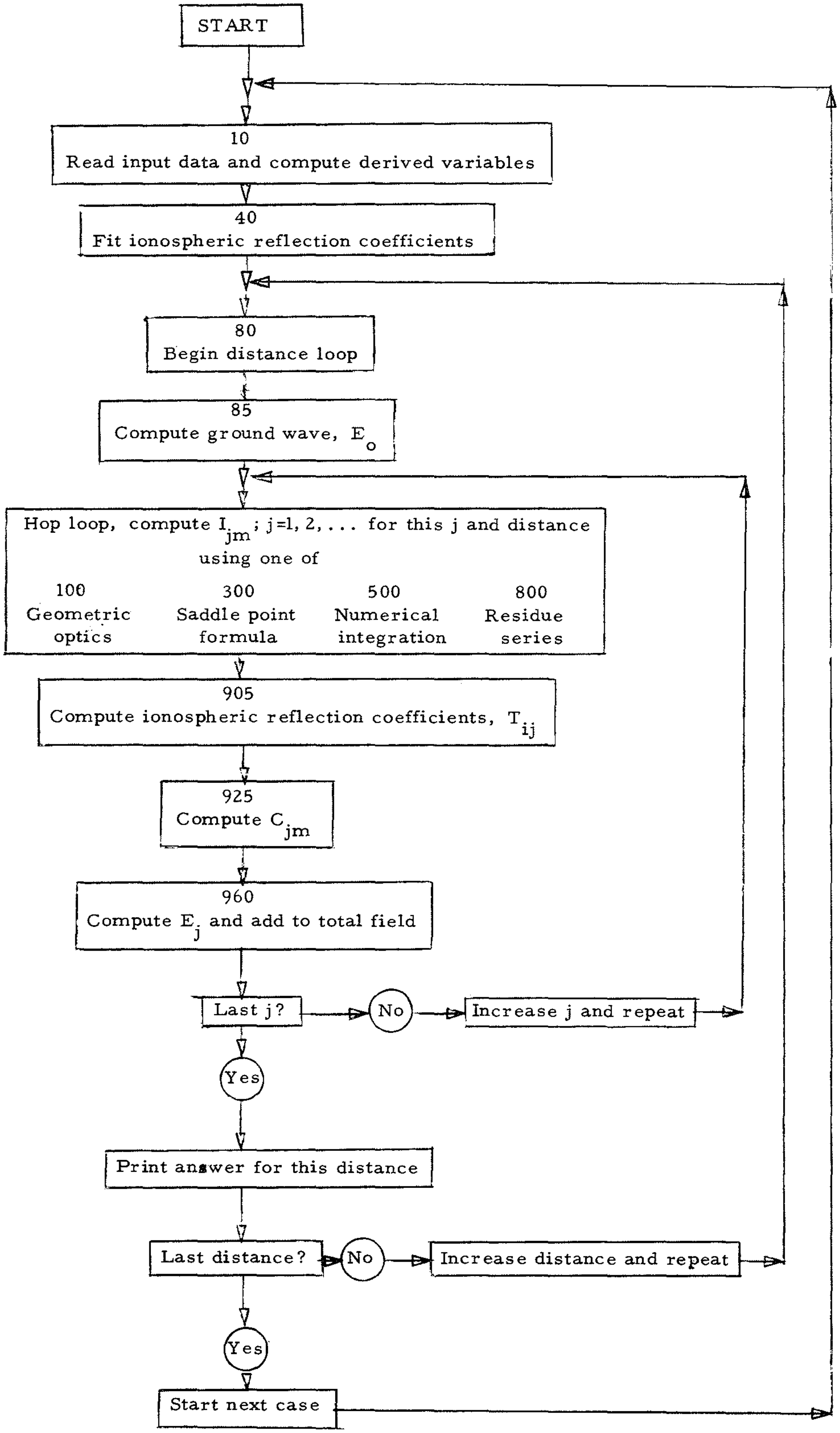

Figure 2. Flow chart for Program ANIHOP.

first entry to LPAINR. The b_i (43) are also independent of distance and are computed and stored on this first entry. Then, on every entry, e^{-ixt} and its derivatives are computed and the derivatives of the product $e^{-ixt}(A(t)/e^{-ixt}) = A(t)$ are computed using (41). The residues are then computed with (38).

The storage in LPAINR is set up for at most 5 hops since the sixth hop would not normally use LPAINR except for paths longer than 10,000 km. Because quantities (such as the b_i) need to be stored only for $M \leq j$ (and for $M > 1$ for the q_m poles), we save some storage by overlaying the arrays for the q_m poles on those for the q_e poles. If necessary, the details can be determined by examining the FORTRAN listing.

4.2 Input Data and a Test Case

Table 1 lists the input data necessary for each case, and figure 3 shows the arrangement of an input data deck. You can stack as many cases in sequence as you want. Execution terminates when an end-of-file card is read by statement 10.

Input data for a test case are listed in figure 4. The reflection coefficients used in this test case were computed with the program ANIREF, which is described in the appendix.

The printout produced by the test case is listed in figures 5-13. The "PHI C" printed out is the "phase lag" related to the phase of the field by

$$\varphi_c = -[\text{phase }(E) + kd + \pi/2],$$

where phase (E) is the phase of the total field or any of its components, as appropriate.

4.3 Important Compiler Characteristics

In our FORTRAN system, the variable index of a DO loop is compared with the maximum value before the DO loop is executed, and if the index exceeds the maximum value, the loop is skipped. This corresponds with the usual mathematical convention that a sum or product is empty if the lower limit of the index exceeds the upper limit, and we have found it convenient to use this feature in the program and its subroutines. If your compiler tests the index of the DO loop <u>after</u> execution, you will need to modify the program accordingly.

Our computer stores two-dimensional arrays columnwise. That is, if A(i,j) is a two-dimensional array, all of the elements of the column j=1 are stored in sequence, followed by all the elements of the column j=2, etc. A three-dimensional array can be thought of as an ordered sequence of two-dimensional arrays, with the third subscript being the number of the two-dimensional array. Therefore, a three-dimensional array can be used in place of a two-dimensional array by fixing the value of the third subscript. Similarly, a two-dimensional array can be treated as a one-dimensional array by fixing the value of the second subscript. We have used this feature extensively in subroutine LPAINR.

4.4 FORTRAN Listing of PROGRAM ANIHOP and Its Subroutines

See FORTRAN listing of ANIHOP beginning on page 33.

5. ACKNOWLEDGMENTS

Revision of the propagation program was sponsored by the Defense Atomic Support Agency and monitored by Mr. Dow Evelyn. Mrs. Mary Chrisman wrote the early versions of the program and several of its subroutines. Dr. George Hufford originated the AIRY function subroutine. The reflection coefficient program in the appendix incorporates many improvements developed by Dr. Bernard Wieder.

We would especially like to thank Mr. David Morfitt of NWC, Corona; Dr. Gary Price of SRI; Mr. Burt Gambill of GE Tempo; and Dr. G. Gonzales of University of Miami for many suggestions and discussions of the earlier version of the program.

Table 1. Input Data for the Program ANIHOP

Variable	Physical Quantity	Units	Comments
CNDK		mho/m	Ground conductivity of path.
EPS2			Ground dielectric constant relative to free space.
DMIN	Distance	km	Minimum distance at which field is calculated.
DELTA	Distance	km	Distance increment.
DMAX	Distance	km	Largest distance for which NHOP hops will be used.
NHOP	Number of hops		See preceding comment.
NT	Number		Reflection coefficients are read in for this many angles of incidence.
POWER	Radiated power	watts	CCIR definition.
KASE			Controls punching of output cards; 1 means punch cards; 0 means no cards output.
ICOND	Number		Number of distance blocks to follow. May be 0.
XD(I)	Distance	km	NH(I) hops will be calculated for XD(I-1) to XD(I), where NH(0) = NHOP, and XD(0) = DMAX.
NH(I)	Number of hops		
ID(I), ITIM, IPHI, PHIA	Various (see comments)		Identification of ionospheric reflection coefficients; such as time, magnetic field, azimuth, etc.
FREQ	Radio frequency	kHz	

Table 1. Input Data for the Program ANIHOP (continued)

Variable	Physical Quantity	Units	Comments
PHI(N)		Degrees	Angle of incidence on ionosphere
TAMP(L, N)	T		Ionospheric reflection coefficient for PHI(N): L = 1 for T_{ee}, L = 2 for T_{em}, L = 3 for T_{me}, L = 4 for T_{mm}.
TPHA(L, N)	Phase of T	radians	
HBOT	Height	km	The input phase(T) is referenced to this height.
HREF	Height	km	Effective height of reflection--phase(T) will re-referenced to this height in program.

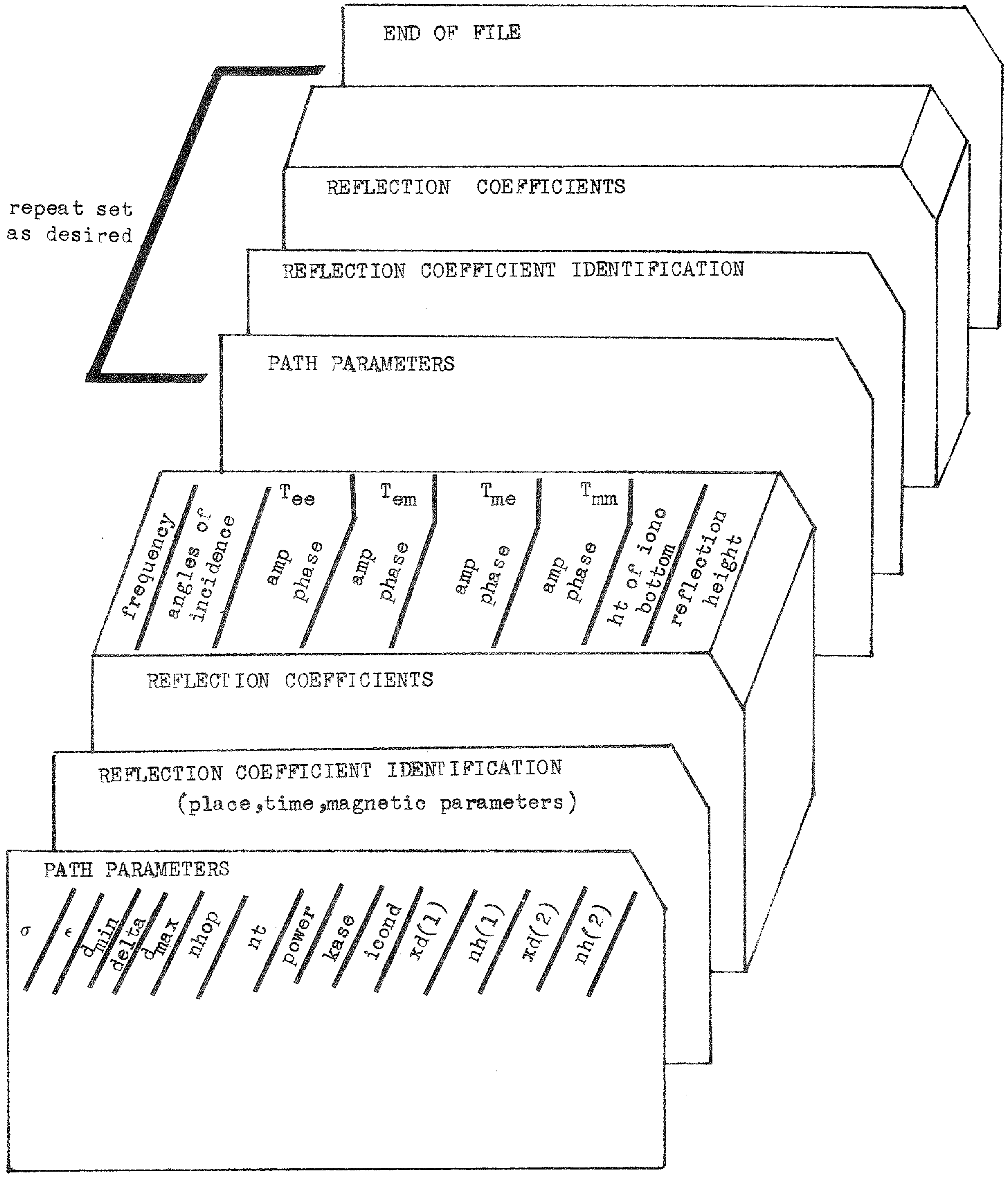

Figure 3. Data deck set-up for program Anihop. The top line on the dummy card is a descriptive label; it should not be punched on the card.

ANIHOP INPUT

σ / ϵ / dmin / delta / dmax / nhop / nt / power / kase / icond / xd(1) / nh(1) / xd(2) / nh(2) / xd(3) / nh(3)

```
  4.  80.  600.  200.  1000. 2  6  1000. 0 3  6000. 5  8000. 7 10000. 9
SAN FRAN SEAPATH   NIGHT        PHIA= 213.5 DIP=   6.5 HM= 0.310 GAUSS
  30.065.02.386-001 1.9271.830-001-1.0187.001-002 1.5413.332-001-1.049 74.0 76.3
  30.073.01.627-001 2.8252.093-001 0.9357.233-002-1.8514.838-001 0.565 74.0 74.9
  30.078.01.527-001 2.5762.187-001 1.8929.881-002-0.7885.981-001 1.402 74.0 76.2
  30.080.02.031-001 2.4752.135-001 2.2311.028-001-0.4436.503-001 1.713 74.0 77.1
  30.081.02.410-001 2.4752.081-001 2.3921.027-001-0.2816.781-001 1.865 74.0 77.4
  30.082.02.866-001 2.5042.005-001 2.5481.010-001-0.1257.073-001 2.014 74.0 77.6
  4.  80.  600.  200.  1000. 2  6  1000. 0 3  6000. 5  8000. 7 10000. 9
SAN FRAN SEAPATH   NIGHT        PHIA= 258.1 DIP=  39.0 HM= 0.370 GAUSS
  30.065.03.816-001 1.1412.662-001-1.1152.343-001-0.8282.081-001 0.630 74.0 77.8
  30.073.02.596-001 2.6713.786-001 0.7353.404-001 0.7832.047-001 1.153 74.0 75.3
  30.078.01.394-001 2.9984.035-001 1.6713.596-001 1.6513.211-001 1.513 74.0 74.6
  30.080.01.270-001 2.6723.977-001 2.0173.533-001 1.9793.909-001 1.730 74.0 76.2
  30.081.01.464-001 2.4953.902-001 2.1853.462-001 2.1404.303-001 1.849 74.0 77.3
  30.082.01.827-001 2.4003.791-001 2.3513.360-001 2.2994.729-001 1.974 74.0 78.2
  4.  80.  600.  200.  1000. 2  6  1000. 0 3  6000. 5  8000. 7 10000. 9
SAN FRAN SEAPATH   NIGHT        PHIA= 299.2 DIP=  66.9 HM= 0.510 GAUSS
  30.065.03.090-001 0.4292.224-001-1.1422.389-001-1.4034.374-001 1.632 74.0 79.1
  30.073.03.213-001 1.4283.384-001 0.7743.682-001 0.7523.215-001-2.103 74.0 78.7
  30.078.03.986-001 1.8263.487-001 1.7753.877-001 1.8133.480-001-0.338 74.0 79.0
  30.080.04.569-001 1.9983.357-001 2.1563.755-001 2.2094.015-001 0.321 74.0 79.2
  30.081.04.922-001 2.0913.247-001 2.3453.642-001 2.4044.371-001 0.633 74.0 79.3
  30.082.05.316-001 2.1893.103-001 2.5323.489-001 2.5974.782-001 0.933 74.0 79.4
END OF FILE
```

← **identification**

frequency / angle of incidence / amp phase (T_{ee}) / amp phase (T_{em}) / amp phase (T_{me}) / amp phase (T_{mm}) / ht of iono bottom / reference height

Figure 4. List of input data deck for ANIHOP sample case.

```
            WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR      1000  WATTS RADIATED POWER

          FREQUENCY =    30.00KHZ, EARTH CONDUCTIVITY =  4.0000 MHO/M,  RELATIVE PERMITIVITY =  80.00

                                        IONOSPHERIC MODEL


SAN FRAN SEAPATH   NIGHT       PHIA= 213.5 DIP=   6.5 HM= 0.310 GAUSS          REFERENCE HEIGHT =  77.600

    PHI  COS(PHI)       T E E               T E M               T M E               T M M            HBOT     HREF
                  AMPLITUDE  PHASE    AMPLITUDE  PHASE    AMPLITUDE  PHASE    AMPLITUDE  PHASE
  65.00  0.4226   0.23860     3.840   0.18300     0.895   0.07001    -2.829   0.33320     0.864      74.0      76.3
  73.00  0.2924   0.16270     4.149   0.20930     2.259   0.07233    -0.527   0.48380     1.889      74.0      74.9
  78.00  0.2079   0.15270     3.517   0.21870     2.833   0.09881     0.153   0.59810     2.343      74.0      76.2
  80.00  0.1736   0.20310     3.261   0.21350     3.017   0.10280     0.343   0.65030     2.499      74.0      77.1
  81.00  0.1564   0.24100     3.183   0.20810     3.100   0.10270     0.427   0.67810     2.573      74.0      77.4
  82.00  0.1392   0.28660     3.134   0.20050     3.178   0.10100     0.505   0.70730     2.644      74.0      77.6

 DISTANCE        TOTAL     GR WAVE   HOP 1     HOP 2     HOP 3     HOP 4     HOP 5     HOP 6     HOP 7     HOP 8     HOP 9

     600 AMP     2.75-004  3.44-004  1.58-004  8.31-005
         PHI C  -0.36      0.40     -1.93     -1.61

     800 AMP     1.99-004  2.08-004  1.30-004  3.06-005
         PHI C   1.09      0.59      2.58     -0.28

    1000 AMP     2.40-004  1.32-004  1.27-004  1.18-005
         PHI C   1.08      0.78      1.48     -0.75

    1200 AMP     2.07-004  8.50-005  1.28-004  5.27-006  6.56-006  5.85-006  6.72-139
         PHI C   1.02      0.98      0.93      3.09      2.42     -3.00     -0.03

    1400 AMP     1.88-004  5.57-005  1.24-004  1.17-005  3.45-006  2.96-006  3.35-006
         PHI C   0.84      1.19      0.62      1.40      0.66      2.87      0.83

    1600 AMP     1.45-004  3.68-005  1.13-004  1.48-005  2.51-006  1.93-006  1.32-006
         PHI C   0.57      1.39      0.48     -0.60      0.41     -1.21     -1.25

    1800 AMP     9.50-005  2.45-005  9.88-005  1.68-005  3.63-006  1.49-006  8.53-007
         PHI C   0.60      1.59      0.44     -2.04      2.17      3.01     -0.91

    2000 AMP     7.35-005  1.65-005  8.35-005  1.86-005  4.36-006  1.40-006  6.60-007
         PHI C   0.76      1.80      0.47     -3.06     -0.98      2.55      1.48

    2200 AMP     7.12-005  1.11-005  7.06-005  1.95-005  4.56-006  1.47-006  5.94-007
         PHI C   1.16      2.00      0.70      2.49     -2.88     -2.82     -0.71

    2400 AMP     7.66-005  7.51-006  5.76-005  1.97-005  4.52-006  1.51-006  5.86-007
         PHI C   1.20      2.21      0.87      1.95      1.33     -0.55     -1.47

    2600 AMP     6.54-005  5.11-006  4.56-005  1.94-005  4.37-006  1.45-006  5.97-007
         PHI C   1.18      2.41      1.01      1.53     -0.30      2.42     -1.08
```

Figure 5. Printed output from ANIHOP sample case, page 1.

```
2800 AMP    5.27-005   3.48-006   3.57-005   1.90-005   4.21-006   1.39-006   5.96-007
     PHI C  1.18       2.62       1.16       1.21      -1.52       0.42       0.35

3000 AMP    4.09-005   2.38-006   2.76-005   1.86-005   4.05-006   1.30-006   5.72-007
     PHI C  1.31       2.82       1.32       0.97      -2.48      -2.06       2.64

3200 AMP    3.65-005   1.63-006   2.11-005   1.82-005   3.95-006   1.22-006   5.54-007
     PHI C  1.34       3.03       1.49       0.81       3.03       2.12      -0.82

3400 AMP    3.19-005   1.12-006   1.60-005   1.76-005   3.89-006   1.18-006   5.50-007
     PHI C  1.31      -3.05       1.67       0.72       2.41       0.33       2.45

3600 AMP    2.65-005   7.70-007   1.21-005   1.68-005   3.92-006   1.11-006   5.71-007
     PHI C  1.25      -2.85       1.85       0.69       1.91      -1.16       0.78

3800 AMP    2.18-005   5.30-007   9.08-006   1.55-005   4.10-006   1.03-006   5.89-007
     PHI C  1.26      -2.64       2.04       0.70       1.50      -2.41      -1.60

4000 AMP    2.07-005   3.66-007   6.78-006   1.43-005   4.40-006   9.78-007   6.12-007
     PHI C  1.28      -2.44       2.23       0.77       1.19       2.80       2.63

4200 AMP    2.06-005   2.53-007   5.04-006   1.34-005   4.77-006   9.44-007   6.37-007
     PHI C  1.25      -2.23       2.42       0.98       0.95       1.90       0.85

4400 AMP    1.82-005   1.75-007   3.73-006   1.23-005   5.13-006   9.31-007   6.57-007
     PHI C  1.27      -2.03       2.61       1.22       0.80       1.14      -0.70

4600 AMP    1.54-005   1.21-007   2.76-006   1.10-005   5.42-006   9.34-007   6.64-007
     PHI C  1.36      -1.82       2.81       1.47       0.70       0.53      -2.02

4800 AMP    1.30-005   8.42-008   2.03-006   9.18-006   5.59-006   9.36-007   6.73-007
     PHI C  1.39      -1.62       3.00       1.61       0.66       0.05       3.12

5000 AMP    1.12-005   5.84-008   1.49-006   7.59-006   5.63-006   9.27-007   6.86-007
     PHI C  1.39      -1.41      -3.08       1.77       0.65      -0.31       2.12

5200 AMP    9.32-006   4.06-008   1.10-006   6.20-006   5.52-006   9.44-007   7.01-007
     PHI C  1.35      -1.21      -2.88       1.93       0.68      -0.58       1.25

5400 AMP    7.42-006   2.83-008   8.02-007   5.02-006   5.28-006   1.01-006   7.17-007
     PHI C  1.29      -1.00      -2.69       2.10       0.73      -0.74       0.50

5600 AMP    5.66-006   1.97-008   5.86-007   4.02-006   4.95-006   1.12-006   7.33-007
     PHI C  1.23      -0.80      -2.49       2.28       0.80      -0.79      -0.14

5800 AMP    4.19-006   1.37-008   4.28-007   3.20-006   4.53-006   1.27-006   7.55-007
     PHI C  1.18      -0.59      -2.29       2.46       0.89      -0.75      -0.69

6000 AMP    3.16-006   9.57-009   3.12-007   2.53-006   4.16-006   1.45-006   7.80-007
     PHI C  1.13      -0.39      -2.09       2.64       1.01      -0.65      -1.15

6200 AMP    2.90-006   6.68-009   2.27-007   1.98-006   3.92-006   1.63-006   7.98-007   3.70-007   1.85-007
     PHI C  1.10      -0.18      -1.89       2.82       1.20      -0.52      -1.53       0.74      -0.78

6400 AMP    2.18-006   4.67-009   1.65-007   1.55-006   3.64-006   1.79-006   8.11-007   3.78-007   1.88-007
     PHI C  1.22       0.02      -1.69       3.01       1.42      -0.37      -1.83      -0.02      -2.08
```

Figure 6. Printed output from ANIHOP sample case, page 2.

```
 6600 AMP     1.80-006   3.26-009   1.20-007   1.20-006   3.33-006   1.92-006   8.28-007   3.87-007   1.91-007
      PHI C   1.51       0.22      -1.49      -3.09       1.66      -0.20      -2.09      -0.69       3.03

 6800 AMP     1.64-006   2.28-009   8.74-008   9.31-007   3.00-006   2.00-006   8.61-007   3.96-007   1.95-007
      PHI C   1.82       0.43      -1.28      -2.90       1.91      -0.04      -2.28      -1.28       1.96

 7000 AMP     1.40-006   1.60-009   6.34-008   7.17-007   2.58-006   2.02-006   9.08-007   4.07-007   1.99-007
      PHI C   2.01       0.63      -1.08      -2.71       2.11       0.13      -2.39      -1.80       0.99

 7200 AMP     1.16-006   1.12-009   4.60-008   5.50-007   2.17-006   1.99-006   9.55-007   4.23-007   2.04-007
      PHI C   2.22       0.84      -0.88      -2.51       2.29       0.30      -2.41      -2.24       0.12

 7400 AMP     1.07-006   7.84-010   3.33-008   4.21-007   1.81-006   1.91-006   9.87-007   4.41-007   2.08-007
      PHI C   2.52       1.04      -0.68      -2.32       2.46       0.46      -2.36      -2.61      -0.66

 7600 AMP     1.10-006   5.50-010   2.42-008   3.21-007   1.50-006   1.79-006   9.99-007   4.58-007   2.13-007
      PHI C   2.77       1.25      -0.48      -2.13       2.64       0.63      -2.23      -2.91      -1.36

 7800 AMP     1.14-006   3.86-010   1.75-008   2.44-007   1.23-006   1.65-006   9.89-007   4.72-007   2.20-007
      PHI C   2.90       1.45      -0.27      -1.93       2.82       0.80      -2.05       3.13      -1.99

 8000 AMP     1.10-006   2.71-010   1.27-008   1.85-007   1.00-006   1.52-006   9.66-007   4.90-007   2.27-007
      PHI C   2.90       1.66      -0.07      -1.73       3.00       0.98      -1.83       2.92      -2.54

 8200 AMP     9.70-007   1.90-010   9.17-009   1.40-007   8.10-007   1.41-006   9.35-007   5.17-007   2.33-007   1.18-007   5.91-008
      PHI C   2.86       1.86       0.13      -1.54      -3.10       1.20      -1.57       2.76      -3.03      -0.48      -2.21

 8400 AMP     1.02-006   1.34-010   6.63-009   1.06-007   6.52-007   1.30-006   9.04-007   5.56-007   2.41-007   1.22-007   6.10-008
      PHI C   2.73       2.07       0.33      -1.34      -2.92       1.44      -1.29       2.66       2.82      -1.27       2.88

 8600 AMP     1.06-006   9.39-011   4.80-009   7.99-008   5.23-007   1.18-006   8.73-007   6.02-007   2.50-007   1.25-007   6.29-008
      PHI C   2.58       2.27       0.54      -1.14      -2.73       1.69      -1.01       2.62       2.45      -1.99       1.77

 8800 AMP     1.07-006   6.61-011   3.47-009   6.01-008   4.17-007   1.07-006   8.42-007   6.44-007   2.62-007   1.29-007   6.49-008
      PHI C   2.49       2.48       0.74      -0.94      -2.54       1.95      -0.72       2.66       2.14      -2.65       0.75

 9000 AMP     1.06-006   4.65-011   2.51-009   4.52-008   3.31-007   9.56-007   8.10-007   6.74-007   2.72-007   1.33-007   6.70-008
      PHI C   2.47       2.68       0.94      -0.75      -2.35       2.22      -0.45       2.75       1.90       3.04      -0.20

 9200 AMP     1.00-006   3.27-011   1.81-009   3.39-008   2.62-007   8.40-007   7.72-007   6.86-007   2.80-007   1.38-007   6.92-008
      PHI C   2.49       2.89       1.14      -0.55      -2.16       2.48      -0.19       2.89       1.70       2.51      -1.07

 9400 AMP     9.18-007   2.31-011   1.31-009   2.54-008   2.07-007   7.15-007   7.28-007   6.78-007   2.89-007   1.43-007   7.13-008
      PHI C   2.47       3.09       1.35      -0.35      -1.97       2.67       0.07       3.07       1.53       2.04      -1.88

 9600 AMP     8.11-007   1.63-011   9.46-010   1.90-008   1.62-007   6.03-007   6.79-007   6.52-007   3.03-007   1.48-007   7.37-008
      PHI C   2.43      -2.99       1.55      -0.15      -1.78       2.87       0.31      -3.01       1.39       1.63      -2.62

 9800 AMP     6.76-007   1.15-011   6.84-010   1.42-008   1.27-007   5.06-007   6.24-007   6.14-007   3.26-007   1.53-007   7.64-008
      PHI C   2.34      -2.78       1.75       0.05      -1.58       3.06       0.54      -2.77       1.31       1.26       2.99

10000 AMP     5.40-007   8.09-012   4.94-010   1.06-008   9.92-008   4.21-007   5.76-007   5.67-007   3.54-007   1.59-007   7.90-008
      PHI C   2.16      -2.58       1.96       0.25      -1.39      -3.03       0.77      -2.51       1.29       0.93       2.38
```

Figure 7. Printed output from ANIHOP sample case, page 3.

```
            WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR      1000  WATTS RADIATED POWER

          FREQUENCY =    30.00KHZ, EARTH CONDUCTIVITY =  4.0000 MHO/M,  RELATIVE PERMITIVITY =  80.00

                                        IONOSPHERIC MODEL

SAN FRAN SEAPATH   NIGHT       PHIA= 258.1 DIP=  39.0 HM= 0.370 GAUSS         REFERENCE HEIGHT =  78.200

    PHI  COS(PHI)       T E E               T E M               T M E               T M M              HBOT      HREF
                  AMPLITUDE  PHASE    AMPLITUDE  PHASE    AMPLITUDE  PHASE    AMPLITUDE  PHASE
  65.00  0.4226   0.38160     3.373   0.26620     1.117   0.23430     1.404   0.20810     2.862         74.0       77.8
  73.00  0.2924   0.25960     4.215   0.37860     2.279   0.34040     2.327   0.20470     2.697         74.0       75.3
  78.00  0.2079   0.13940     4.096   0.40350     2.769   0.35960     2.749   0.32110     2.611         74.0       74.6
  80.00  0.1736   0.12700     3.589   0.39770     2.934   0.35330     2.896   0.39090     2.647         74.0       76.2
  81.00  0.1564   0.14640     3.321   0.39020     3.011   0.34620     2.966   0.43030     2.675         74.0       77.3
  82.00  0.1392   0.18270     3.135   0.37910     3.086   0.33600     3.034   0.47290     2.709         74.0       78.2

 DISTANCE        TOTAL     GR WAVE   HOP 1     HOP 2     HOP 3     HOP 4     HOP 5     HOP 6     HOP 7     HOP 8     HOP 9

     600 AMP     4.15-004  3.44-004  2.17-004  2.19-004
         PHI C   0.16      0.40     -1.96      0.64

     800 AMP     3.33-004  2.08-004  1.27-004  1.00-004
         PHI C   1.26      0.59      2.22      1.51

    1000 AMP     3.20-004  1.32-004  9.49-005  9.82-005
         PHI C   0.91      0.78      1.16      0.85

    1200 AMP     1.72-004  8.50-005  8.04-005  8.95-005  3.01-005  3.30-005  2.66-139
         PHI C   1.29      0.98      0.72      2.71     -1.15      1.31     -1.89

    1400 AMP     2.06-004  5.57-005  7.55-005  7.79-005  2.97-005  1.21-005  3.30-005
         PHI C   0.55      1.19      0.52      0.14     -2.45      0.34      0.74

    1600 AMP     2.84-005  3.68-005  6.86-005  7.04-005  3.13-005  8.33-006  7.93-006
         PHI C  -1.19      1.39      0.43     -2.13     -1.76      2.81     -2.64

    1800 AMP     1.01-004  2.45-005  5.99-005  6.47-005  2.47-005  7.97-006  4.07-006
         PHI C   1.35      1.59      0.42      2.56      0.36      1.27     -2.39

    2000 AMP     1.06-004  1.65-005  5.08-005  5.78-005  2.03-005  8.77-006  3.34-006
         PHI C   1.23      1.80      0.46      1.44     -2.87      1.33      0.37

    2200 AMP     1.08-004  1.11-005  3.97-005  5.29-005  1.83-005  1.00-005  3.40-006
         PHI C   0.96      2.00      0.78      0.61      1.42      2.59     -1.36

    2400 AMP     7.70-005  7.51-006  3.13-005  4.98-005  1.76-005  7.36-006  3.76-006
         PHI C   0.13      2.21      1.01      0.01     -0.75     -1.29     -1.75

    2600 AMP     3.84-005  5.11-006  2.48-005  4.74-005  1.71-005  6.12-006  4.23-006
         PHI C  -0.02      2.41      1.14     -0.39     -2.44      1.72     -1.07

    2800 AMP     3.55-005  3.48-006  1.94-005  4.51-005  1.54-005  5.50-006  4.31-006
         PHI C   0.10      2.62      1.29     -0.63      2.52     -0.23      0.53
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Figure 8. Printed output from ANIHOP sample case, page 4.

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3000 AMP     2.40-005  2.38-006  1.50-005  4.24-005  1.40-005  5.19-006  3.41-006
     PHI C  -0.02      2.82      1.45     -0.75      1.45     -2.77      2.92

3200 AMP     4.04-005  1.63-006  1.15-005  3.90-005  1.32-005  5.03-006  2.93-006
     PHI C  -0.11      3.03      1.63     -0.77      0.58      1.41     -0.45

3400 AMP     4.17-005  1.12-006  8.74-006  3.50-005  1.28-005  4.98-006  2.64-006
     PHI C  -0.44     -3.05      1.80     -0.73     -0.12     -0.38      2.90

3600 AMP     3.81-005  7.70-007  6.59-006  3.08-005  1.27-005  4.80-006  2.50-006
     PHI C  -0.64     -2.85      1.99     -0.64     -0.68     -1.87      1.23

3800 AMP     3.18-005  5.30-007  4.95-006  2.65-005  1.28-005  4.43-006  2.39-006
     PHI C  -0.80     -2.64      2.17     -0.51     -1.11     -3.11     -1.12

4000 AMP     2.14-005  3.66-007  3.69-006  2.23-005  1.32-005  4.18-006  2.32-006
     PHI C  -0.76     -2.44      2.36     -0.36     -1.43      2.09      3.11

4200 AMP     1.96-005  2.53-007  2.75-006  1.80-005  1.39-005  4.04-006  2.28-006
     PHI C  -0.57     -2.23      2.55     -0.22     -1.63      1.15      1.32

4400 AMP     2.12-005  1.75-007  2.04-006  1.42-005  1.46-005  3.97-006  2.26-006
     PHI C  -0.73     -2.03      2.75     -0.08     -1.73      0.34     -0.26

4600 AMP     2.02-005  1.21-007  1.50-006  1.10-005  1.51-005  4.01-006  2.05-006
     PHI C  -1.03     -1.82      2.94      0.07     -1.75     -0.36     -1.60

4800 AMP     1.77-005  8.42-008  1.11-006  8.59-006  1.51-005  4.15-006  1.89-006
     PHI C  -1.23     -1.62      3.14      0.28     -1.70     -0.95     -2.80

5000 AMP     1.53-005  5.84-008  8.14-007  6.67-006  1.46-005  4.30-006  1.77-006
     PHI C  -1.31     -1.41     -2.95      0.49     -1.61     -1.43      2.42

5200 AMP     1.38-005  4.06-008  5.97-007  5.14-006  1.38-005  4.45-006  1.69-006
     PHI C  -1.30     -1.21     -2.75      0.70     -1.48     -1.83      1.47

5400 AMP     1.29-005  2.83-008  4.37-007  3.94-006  1.26-005  4.73-006  1.64-006
     PHI C  -1.27     -1.00     -2.55      0.92     -1.32     -2.17      0.62

5600 AMP     1.19-005  1.97-008  3.20-007  3.00-006  1.13-005  5.22-006  1.63-006
     PHI C  -1.29     -0.80     -2.36      1.14     -1.15     -2.44     -0.12

5800 AMP     1.05-005  1.37-008  2.34-007  2.28-006  9.92-006  5.91-006  1.63-006
     PHI C  -1.37     -0.59     -2.16      1.36     -0.97     -2.60     -0.78

6000 AMP     9.02-006  9.57-009  1.70-007  1.72-006  8.54-006  6.63-006  1.65-006
     PHI C  -1.52     -0.39     -1.96      1.58     -0.77     -2.68     -1.37

6200 AMP     6.88-006  6.68-009  1.24-007  1.30-006  7.19-006  7.24-006  1.72-006  6.04-007  3.12-007
     PHI C  -1.77     -0.18     -1.76      1.80     -0.56     -2.67     -1.88      1.78      1.70

6400 AMP     6.23-006  4.67-009  9.03-008  9.74-007  5.95-006  7.62-006  1.81-006  5.97-007  2.88-007
     PHI C  -1.88      0.02     -1.56      2.02     -0.35     -2.60     -2.30      0.92      0.35
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Figure 9. Printed output from ANIHOP sample case, page 8.

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 6600 AMP    6.39-006   3.26-009   6.56-008   7.30-007   4.86-006   7.74-006   1.88-006   5.93-007   2.70-007
      PHI C -2.00       0.22      -1.35       2.24      -0.13      -2.49      -2.65       0.13      -0.90

 6800 AMP    6.57-006   2.28-009   4.77-008   5.46-007   3.91-006   7.61-006   1.96-006   5.99-007   2.55-007
      PHI C -2.12       0.43      -1.15       2.46       0.10      -2.35      -2.96      -0.59      -2.05

 7000 AMP    6.33-006   1.60-009   3.46-008   4.07-007   3.13-006   7.26-006   2.10-006   6.21-007   2.45-007
      PHI C -2.20       0.63      -0.95       2.68       0.32      -2.19       3.03      -1.23      -3.11

 7200 AMP    5.65-006   1.12-009   2.51-008   3.04-007   2.49-006   6.75-006   2.38-006   6.43-007   2.39-007
      PHI C -2.25       0.84      -0.75       2.90       0.54      -2.01       2.80      -1.79       2.20

 7400 AMP    4.76-006   7.84-010   1.82-008   2.26-007   1.97-006   6.13-006   2.80-006   6.60-007   2.34-007
      PHI C -2.29       1.04      -0.55       3.12       0.76      -1.82       2.66      -2.30       1.30

 7600 AMP    3.92-006   5.50-010   1.32-008   1.68-007   1.54-006   5.46-006   3.27-006   6.89-007   2.34-007
      PHI C -2.37       1.25      -0.35      -2.94       0.98      -1.62       2.62      -2.76       0.47

 7800 AMP    3.34-006   3.86-010   9.55-009   1.25-007   1.21-006   4.79-006   3.71-006   7.48-007   2.38-007
      PHI C -2.52       1.45      -0.14      -2.73       1.20      -1.42       2.65       3.12      -0.28

 8000 AMP    3.11-006   2.71-010   6.91-009   9.24-008   9.38-007   4.13-006   4.05-006   8.16-007   2.42-007
      PHI C -2.70       1.66       0.06      -2.51       1.41      -1.21       2.74       2.81      -0.97

 8200 AMP    3.18-006   1.90-010   5.00-009   6.85-008   7.26-007   3.53-006   4.25-006   8.66-007   2.48-007   9.79-008   4.92-008
      PHI C -2.90       1.86       0.26      -2.29       1.63      -0.99       2.86       2.54      -1.61       2.56       2.27

 8400 AMP    3.10-006   1.34-010   3.62-009   5.07-008   5.60-007   2.96-006   4.32-006   9.04-007   2.63-007   9.65-008   4.62-008
      PHI C -3.00       2.07       0.46      -2.07       1.85      -0.77       3.02       2.28      -2.20       1.64       1.00

 8600 AMP    2.97-006   9.39-011   2.62-009   3.75-008   4.30-007   2.44-006   4.26-006   9.77-007   2.81-007   9.68-008   4.38-008
      PHI C -3.03       2.27       0.67      -1.85       2.07      -0.55      -3.09       2.02      -2.70       0.79      -0.20

 8800 AMP    2.75-006   6.61-011   1.89-009   2.77-008   3.30-007   1.99-006   4.08-006   1.14-006   2.89-007   9.67-008   4.21-008
      PHI C -3.03       2.48       0.87      -1.63       2.29      -0.32      -2.90       1.79      -3.14       0.01      -1.32

 9000 AMP    2.44-006   4.65-011   1.37-009   2.05-008   2.52-007   1.60-006   3.83-006   1.39-006   2.98-007   9.81-008   4.06-008
      PHI C -3.04       2.68       1.07      -1.41       2.50      -0.10      -2.70       1.66       2.71      -0.73      -2.37

 9200 AMP    2.06-006   3.27-011   9.89-010   1.51-008   1.92-007   1.27-006   3.51-006   1.69-006   3.22-007   1.03-007   3.98-008
      PHI C -3.12       2.89       1.28      -1.20       2.72       0.13      -2.50       1.63       2.30      -1.40       2.92

 9400 AMP    1.70-006   2.31-011   7.15-010   1.11-008   1.46-007   1.01-006   3.17-006   1.99-006   3.65-007   1.07-007   3.96-008
      PHI C  3.00       3.09       1.48      -0.98       2.94       0.35      -2.28       1.68       1.96      -2.01       2.00

 9600 AMP    1.50-006   1.63-011   5.17-010   8.22-009   1.11-007   8.05-007   2.81-006   2.25-006   4.09-007   1.10-007   3.92-008
      PHI C  2.77      -2.99       1.68      -0.76      -3.13       0.58      -2.07       1.79       1.72      -2.58       1.14

 9800 AMP    1.47-006   1.15-011   3.73-010   6.06-009   8.40-008   6.36-007   2.46-006   2.45-006   4.37-007   1.16-007   3.96-008
      PHI C  2.56      -2.78       1.89      -0.54      -2.91       0.81      -1.85       1.93       1.52      -3.14       0.31

10000 AMP    1.50-006   8.09-012   2.70-010   4.46-009   6.36-008   5.01-007   2.14-006   2.56-006   4.51-007   1.26-007   4.10-008
      PHI C  2.45      -2.58       2.09      -0.33      -2.70       1.03      -1.63       2.09       1.30       2.67      -0.44
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Figure 10. Printed output from ANIHOP sample case, page 9.

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            WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD STRENGTH (V/M) FOR      1000  WATTS RADIATED POWER

          FREQUENCY =    30.00KHZ, EARTH CONDUCTIVITY =  4.0000 MHO/M,  RELATIVE PERMITIVITY =  80.00

                                        IONOSPHERIC MODEL

SAN FRAN SEAPATH   NIGHT       PHIA= 299.2 DIP=  66.9 HM= 0.510 GAUSS          REFERENCE HEIGHT =  79.400

    PHI  COS(PHI)       T E E               T E M               T M E               T M M            HBOT      HREF
                  AMPLITUDE  PHASE    AMPLITUDE  PHASE    AMPLITUDE  PHASE    AMPLITUDE  PHASE
  65.00  0.4226   0.30900     3.299   0.22240     1.728   0.23890     1.467   0.43740    -1.781      74.0      79.1
  73.00  0.2924   0.32130     3.413   0.33840     2.759   0.36820     2.737   0.32150    -0.118      74.0      78.7
  78.00  0.2079   0.39860     3.238   0.34870     3.187   0.38770     3.225   0.34800     1.074      74.0      79.0
  80.00  0.1736   0.45690     3.177   0.33570     3.335   0.37550     3.388   0.40150     1.500      74.0      79.2
  81.00  0.1564   0.49220     3.153   0.32470     3.407   0.36420     3.466   0.43710     1.695      74.0      79.3
  82.00  0.1392   0.53160     3.134   0.31030     3.477   0.34890     3.542   0.47820     1.878      74.0      79.4

 DISTANCE        TOTAL     GR WAVE   HOP 1     HOP 2     HOP 3     HOP 4     HOP 5     HOP 6     HOP 7     HOP 8     HOP 9

     600 AMP     5.83-004  3.44-004  3.41-004  9.51-005
         PHI C   0.01      0.40     -0.72      1.48

     800 AMP     2.49-004  2.08-004  3.29-004  1.06-004
         PHI C   2.90      0.59     -2.86      2.80

    1000 AMP     4.50-004  1.32-004  3.05-004  9.15-005
         PHI C   1.79      0.78      2.10      1.99

    1200 AMP     3.59-004  8.50-005  2.86-004  6.32-005  3.57-005  6.19-006  8.66-140
         PHI C   1.47      0.98      1.42     -2.58      1.41      2.72      2.90

    1400 AMP     3.72-004  5.57-005  2.62-004  3.67-005  4.24-005  5.09-006  5.99-007
         PHI C   0.96      1.19      1.05      0.84      0.00     -2.94      1.47

    1600 AMP     3.13-004  3.68-005  2.33-004  2.01-005  4.32-005  1.04-005  1.94-006
         PHI C   0.79      1.39      0.88     -0.75      0.48     -0.04      0.33

    1800 AMP     2.04-004  2.45-005  2.01-004  1.05-005  3.90-005  1.47-005  2.10-006
         PHI C   1.03      1.59      0.82     -1.67      2.41     -1.69      1.36

    2000 AMP     1.45-004  1.65-005  1.69-004  1.36-005  3.28-005  1.70-005  3.41-006
         PHI C   0.61      1.80      0.84     -2.01     -0.94     -1.81     -1.71

    2200 AMP     1.12-004  1.11-005  1.41-004  1.73-005  2.78-005  1.79-005  5.08-006
         PHI C   1.19      2.00      0.97     -2.58     -3.04     -0.74      2.82

    2400 AMP     1.56-004  7.51-006  1.15-004  1.75-005  2.47-005  1.76-005  6.31-006
         PHI C   1.30      2.21      1.11     -3.13      0.92      1.40      2.30

    2600 AMP     6.87-005  5.11-006  9.12-005  1.53-005  2.25-005  1.65-005  6.99-006
         PHI C   1.39      2.41      1.25      2.54     -0.94     -2.01      2.81

    2800 AMP     8.33-005  3.48-006  7.15-005  1.50-005  2.07-005  1.52-005  7.20-006
         PHI C   1.70      2.62      1.40      1.76     -2.46      1.41     -2.10
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Figure 11. Printed output from ANIHOP sample case, page 7.

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3000 AMP    8.24-005  2.38-006  5.54-005  2.02-005  1.93-005  1.44-005  7.19-006
     PHI C  1.44      2.82      1.56      1.16      2.59     -0.32      0.20

3200 AMP    7.50-005  1.63-006  4.25-005  2.80-005  1.85-005  1.35-005  6.86-006
     PHI C  1.68      3.03      1.73      0.88      1.59     -2.47      3.02

3400 AMP    8.41-005  1.12-006  3.23-005  3.49-005  1.80-005  1.28-005  6.38-006
     PHI C  1.25     -3.05      1.91      0.79      0.79      1.99      0.02

3600 AMP    6.67-005  7.70-007  2.44-005  3.96-005  1.72-005  1.23-005  6.02-006
     PHI C  0.88     -2.85      2.09      0.80      0.16      0.44     -1.62

3800 AMP    4.89-005  5.30-007  1.83-005  4.17-005  1.61-005  1.19-005  5.53-006
     PHI C  0.80     -2.64      2.28      0.87     -0.34     -0.88      2.26

4000 AMP    3.24-005  3.66-007  1.37-005  4.13-005  1.51-005  1.16-005  5.08-006
     PHI C  0.75     -2.44      2.46      0.97     -0.77     -2.00      0.17

4200 AMP    2.00-005  2.53-007  1.02-005  3.83-005  1.49-005  1.14-005  4.69-006
     PHI C  1.49     -2.23      2.66      1.16     -1.07     -2.97     -1.66

4400 AMP    3.02-005  1.75-007  7.54-006  3.43-005  1.52-005  1.13-005  4.35-006
     PHI C  1.88     -2.03      2.85      1.39     -1.23      2.49      3.02

4600 AMP    3.12-005  1.21-007  5.57-006  3.00-005  1.54-005  1.12-005  4.10-006
     PHI C  1.72     -1.82      3.04      1.63     -1.23      1.79      1.63

4800 AMP    2.38-005  8.42-008  4.11-006  2.63-005  1.54-005  1.11-005  3.93-006
     PHI C  1.37     -1.62     -3.04      1.80     -1.10      1.18      0.42

5000 AMP    1.63-005  5.84-008  3.02-006  2.26-005  1.54-005  1.13-005  3.80-006
     PHI C  0.90     -1.41     -2.85      1.97     -0.87      0.66     -0.65

5200 AMP    1.28-005  4.06-008  2.22-006  1.90-005  1.55-005  1.15-005  3.73-006
     PHI C  0.43     -1.21     -2.65      2.15     -0.58      0.24     -1.59

5400 AMP    1.20-005  2.83-008  1.62-006  1.58-005  1.59-005  1.15-005  3.71-006
     PHI C  0.20     -1.00     -2.45      2.33     -0.28     -0.10     -2.42

5600 AMP    1.22-005  1.97-008  1.19-006  1.30-005  1.63-005  1.13-005  3.71-006
     PHI C  0.23     -0.80     -2.25      2.52      0.02     -0.38      3.12

5800 AMP    1.26-005  1.37-008  8.67-007  1.05-005  1.66-005  1.11-005  3.78-006
     PHI C  0.35     -0.59     -2.05      2.71      0.30     -0.64      2.46

6000 AMP    1.30-005  9.57-009  6.32-007  8.45-006  1.65-005  1.13-005  3.91-006
     PHI C  0.43     -0.39     -1.85      2.90      0.57     -0.85      1.89

6200 AMP    1.29-005  6.68-009  4.60-007  6.73-006  1.52-005  1.19-005  4.05-006  1.34-006  1.21-006
     PHI C  0.26     -0.18     -1.65      3.10      0.83     -1.00      1.39     -1.08     -0.42

6400 AMP    8.95-006  4.67-009  3.35-007  5.32-006  1.36-005  1.28-005  4.18-006  1.27-006  1.16-006
     PHI C  0.09      0.02     -1.45     -2.99      1.09     -1.06      0.94     -1.96     -1.87
```

Figure 12. Printed output from ANIHOP sample case, page 5.

```
 6600 AMP      6.99-006   3.26-009   2.44-007   4.18-006   1.19-005   1.36-005   4.39-006   1.23-006   1.12-006
      PHI C    0.00       0.22      -1.25      -2.79       1.37      -1.04       0.53      -2.75       3.10

 6800 AMP      8.00-006   2.28-009   1.77-007   3.27-006   1.02-005   1.42-005   4.76-006   1.21-006   1.09-006
      PHI C   -0.12       0.43      -1.05      -2.60       1.65      -0.95       0.18       2.84       1.90

 7000 AMP      1.06-005   1.60-009   1.29-007   2.54-006   8.89-006   1.44-005   5.20-006   1.20-006   1.06-006
      PHI C   -0.25       0.63      -0.85      -2.40       1.93      -0.80      -0.09       2.24       0.80

 7200 AMP      1.28-005   1.12-009   9.32-008   1.97-006   8.03-006   1.43-005   5.54-006   1.19-006   1.04-006
      PHI C   -0.31       0.84      -0.65      -2.20       2.17      -0.62      -0.29       1.69      -0.21

 7400 AMP      1.36-005   7.84-010   6.76-008   1.51-006   7.12-006   1.39-005   5.70-006   1.24-006   1.03-006
      PHI C   -0.32       1.04      -0.45      -2.00       2.41      -0.40      -0.46       1.21      -1.13

 7600 AMP      1.31-005   5.50-010   4.90-008   1.16-006   6.21-006   1.32-005   5.75-006   1.32-006   1.02-006
      PHI C   -0.28       1.25      -0.24      -1.80       2.63      -0.17      -0.62       0.82      -1.96

 7800 AMP      1.20-005   3.86-010   3.55-008   8.90-007   5.35-006   1.25-005   5.88-006   1.39-006   1.01-006
      PHI C   -0.19       1.45      -0.04      -1.60       2.86       0.08      -0.78       0.50      -2.72

 8000 AMP      1.09-005   2.71-010   2.57-008   6.79-007   4.55-006   1.16-005   6.27-006   1.45-006   1.01-006
      PHI C   -0.09       1.66       0.16      -1.40       3.08       0.33      -0.92       0.22       2.87

 8200 AMP      1.04-005   1.90-010   1.86-008   5.17-007   3.83-006   1.06-005   6.93-006   1.53-006   1.02-006   5.89-007   3.13-007
      PHI C    0.03       1.86       0.36      -1.20      -2.99       0.57      -1.00      -0.07       2.24       0.98       1.48

 8400 AMP      1.03-005   1.34-010   1.35-008   3.92-007   3.19-006   9.51-006   7.73-006   1.72-006   1.03-006   5.88-007   3.04-007
      PHI C   -0.06       2.07       0.57      -1.00      -2.77       0.81      -1.01      -0.33       1.68       0.06       0.15

 8600 AMP      9.89-006   9.39-011   9.73-009   2.97-007   2.64-006   8.34-006   8.51-006   1.99-006   1.04-006   5.91-007   2.95-007
      PHI C   -0.21       2.27       0.77      -0.79      -2.56       1.06      -0.96      -0.53       1.19      -0.79      -1.08

 8800 AMP      9.48-006   6.61-011   7.04-009   2.24-007   2.16-006   7.18-006   9.13-006   2.29-006   1.04-006   5.94-007   2.89-007
      PHI C   -0.33       2.48       0.97      -0.59      -2.34       1.31      -0.85      -0.64       0.73      -1.57      -2.23

 9000 AMP      9.33-006   4.65-011   5.09-009   1.69-007   1.76-006   6.07-006   9.53-006   2.52-006   1.06-006   6.02-007   2.83-007
      PHI C   -0.37       2.68       1.17      -0.39      -2.13       1.57      -0.70      -0.71       0.32      -2.30       2.98

 9200 AMP      9.46-006   3.27-011   3.68-009   1.27-007   1.43-006   5.05-006   9.67-006   2.64-006   1.10-006   6.15-007   2.79-007
      PHI C   -0.33       2.89       1.38      -0.19      -1.92       1.84      -0.53      -0.76      -0.02      -2.97       1.98

 9400 AMP      9.50-006   2.31-011   2.66-009   9.57-008   1.15-006   4.32-006   9.59-006   2.68-006   1.13-006   6.27-007   2.78-007
      PHI C   -0.26       3.09       1.58       0.01      -1.71       2.12      -0.34      -0.83      -0.30       2.71       1.05

 9600 AMP      9.30-006   1.63-011   1.92-009   7.18-008   9.18-007   3.72-006   9.30-006   2.73-006   1.13-006   6.39-007   2.77-007
      PHI C   -0.19      -2.99       1.78       0.22      -1.50       2.40      -0.13      -0.94      -0.54       2.15       0.19

 9800 AMP      8.74-006   1.15-011   1.39-009   5.38-008   7.32-007   3.20-006   8.87-006   2.91-006   1.13-006   6.61-007   2.78-007
      PHI C   -0.14      -2.78       1.99       0.42      -1.29       2.67       0.09      -1.06      -0.77       1.62      -0.61

10000 AMP      7.88-006   8.09-012   1.00-009   4.03-008   5.81-007   2.74-006   8.32-006   3.29-006   1.17-006   6.90-007   2.82-007
      PHI C   -0.12      -2.58       2.19       0.62      -1.08       2.94       0.32      -1.15      -0.99       1.16      -1.36
```

Figure 13. Printed output from ANIHOP sample case, page 6.

6. REFERENCES

Barghausen, A. F., J. W. Finney, and R. M. Fisher (1966), "Radio broadcasting on medium frequencies," unpublished NBS Report 9196.

Belrose, J.S. (1968), "Low and very low frequency radio wave propagation," Radio Wave Propagation (edited by K. Davies), AGARD Lecture Series XXIX, AD-676 788.

Berry, L. A. (1964), "Wave hop theory of long distance propagation of low-frequency radio waves," Radio Science 68D, No. 12.

Berry, L. A. (1967), "Wave hop radio propagation theory," MF, LF, and VLF Radio Propagation, IEE (London) Conference Publication No. 36.

Berry, L. A. and M. E. Chrisman (1965a), "FORTRAN programs for full-wave calculation of LF and VLF radio propagation," unpublished NBS Report 8889.

Berry, L. A. and M. E. Chrisman (1965b), "Numerical values of the path integrals for low and very low frequencies," NBS Tech. Note 319.

Berry, L. A. and M. E. Chrisman (1965c), "The path integrals of LF/VLF wave hop theory," Radio Sci. J. Res. NBS 68D, No. 11.

Berry, L. A., G. Gonzalez, and J. L. Lloyd (1969), "Wave-hop series for an anisotropic ionosphere," Radio Science 4, No. 11.

Berry, L. A. and R. M. Jones (1970), "A time-varying electron density model for LF/VLF propagation calculations, "Telecommunications Tech. Memo. OT/ITSTM 3.

Davies, K. (1965), Ionospheric Radio Propagation, NBS Monograph 80,

Fock, V. A. (1965), Electromagnetic Diffraction and Propagation Problems (Pergamon Press, N. Y.)

Gambill, B. and W. S. Knapp (1969), "A FORTRAN code for the calculation of VLF and LF propagation in a nuclear environment," (WEDCOM), DASA 2273 (General Electric Company, Santa Barbara, Calif.).

Johler, J. R. (1970), "Spherical wave theory for MF, LF,' and VLF propagation, Radio Sci. 5, No. 12.

Johler, J. R. and J. D. Harper, Jr. (1962), "Reflection and transmission of radio waves at a continuously stratified plasma with arbitrary magnetic induction," J. Res. NBS 66D, No. 1.

Kaplan, W. (1952), Advanced Calculus (Addison-Wesley Publishing Co., Inc., Reading, Mass.).

Lewis, R. L. (1971), "The wave hop fields for an inclined dipole over a spherical earth with an anisotropic ionosphere," OT Telecommunications Report

Morfitt, D. G. and R. F. Halley (1970), "Comparison of waveguide and wave hop techniques for VLF propagation modeling," NWC Tech. Publication 4952, Naval Weapons Center, China Lake, California.

Morgan, R. R. (1968), "World-wide VLF effective-conductivity map," Westinghouse Report 80133F-1, submitted to Office of Naval Research, Washington, D.C. 20360.

Pappert, R. A. (1968), "A numerical study of VLF mode structure and polarization below an anisotropic ionosphere," Radio Sci. 4, No. 3.

Sheddy, C., Y. Gough, and R. Pappert (1968), "An improved computer program for VLF mode constants in an earth-ionosphere waveguide of arbitrary electron density distribution," Interim Report No. 682, Naval Electronics Laboratory Center, San Diego, Calif.

Wait, J. R. (1961), "A diffraction theory of LF sky-wave propagation," J. Geophys. Res. 66, No. 6.

Wait, J. R. (1962), Electromagnetic Waves in Stratified Media (Pergamon Press, N.Y.).

Wait, J. R. (1968), "Recent theoretical advances in the terrestrial propagation of VLF electromagnetic waves," (Adv. in Electronics and Electron Physics, Vol. 25, Academic Press Inc., N.Y.).

Watt, A. D. (1967), VLF Radio Engineering (Pergamon Press, N. Y.)

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      PROGRAM ANIHOP
C          A PROGRAM TO COMPUTE LF/VLF RADIO PROPAGATION USING WAVE HOP
C                                THEORY.
C          REFERENCE. A WAVE HOP PROPAGATION PROGRAM FOR AN ANISOTROPIC
C          IONOSPHERE BY L.A. BERRY AND J.E. HERMAN,(ITS, BOULDER, COLO,
C          80302)  TELECOMMUNICATIONS RESEARCH REPORT NO. 11.


      DIMENSION AMP(9),ATI(12),ATR(12),CPHI(13),E(9),FTX(2),G(48)
     1,HOP(9,9),HREF(12),ID(10),JGO(9),NH(4),O(96),PH(9),PHI(12)
     2,PT(45,96),Q(2),R(2,5,5),SD(96),T(40),TA(9,4)
     3,TAMP(4,13),TPHA(4,13),TPR(4,13),TPI(4,13),W(48),XD(4)


      COMMON/PAIN/X,Y,Z,DMIN,XDIST,DSAV,A1,A2,Q,KMAX,T
      COMMON/WGW/QKA1,QKA2,FK,AK1,V,V2, THETA,STH

      COMPLEX B,C,C1,C2,DT1,DT2,E,ETA,F1,F2,F3,GW,HOP,O,PT,Q,QKA2,R,RE,
     1RM,RR,R1,R12,R2,R21,R3,R4,R5,S,SD,SQ2,SUM,S1,S2,T,TA,T1,T2,TY1,TY2



    1 FORMAT(F5.1,F4.1,2F6.1,F7.1,I2,I3,F7.1,2I2,4(F7.1,I2))
    2 FORMAT(2A8,A3,A8,4X,A5,A6,A4,4A8, * REFERENCE HEIGHT =*,F8.3)
    3 FORMAT(F6.1,F4.1,4(E9.3,F6.3),2F5.1)
    4 FORMAT(2A8,A3,A8,4X,A5,A6,A4,4A8)
    5 FORMAT(1H1,10X,* WAVE HOP CALCULATION OF VERTICAL ELECTRIC FIELD S
     1TRENGTH (V/M) FOR  *,F8.0,*  WATTS RADIATED POWER*//10X,
     211HFREQUENCY =    ,F9.2,*KHZ, EARTH CONDUCTIVITY =*,F8.4,* MHO/M,
     3RELATIVE PERMITIVITY =*,F7.2//40X,*IONOSPHERIC MODEL*//)
    7 FORMAT(1H1,8X,5HW(49),15X,5HW(50)/(2E20.8))
    8 FORMAT(1H0F7.0, * AMP    *, 11E10.2)
    9 FORMAT(9X,*PHI C *, F6.2,  10F10.2)

C          ABSCISSA (G) AND WEIGHTS (W)  FOR GAUSSIAN INTEGRATION

      G(1)=-.9987710073  $  G(2)=-.9935301723  $  G(3)=-.9841245837
      G(4)=-.9705915925  $  G(5)=-.9529877032  $  G(6)=-.9313866907
      G(7)=-.9058791367  $  G(8)=-.8765720203  $  G(9)=-.8435882616
      G(10)=-.8070662040  $  G(11)=-.7671590325  $  G(12)=-.7240341309
      G(13)=-.6778723796  $  G(14)=-.6288673968  $  G(15)=-.5772247261
      G(16)=-.5231609747  $  G(17)=-.4669029048  $  G(18)=-.4086864820
      G(19)=-.3487558863  $  G(20)=-.2873624874  $  G(21)=-.2247637904
      G(22)=-.1612223561  $  G(23)=-.09700469921  $  G(24)=-.03238017096
      G(25)=.03238017096  $  G(26)=.09700469921  $  G(27)=.1612223561
      G(28)=.2247637904  $  G(29)=.2873624874  $  G(30)=.3487558863
      G(31)=.4086864820  $  G(32)=.4669029048  $  G(33)=.5231609747
      G(34)=.5772247260  $  G(35)=.6288673968  $  G(36)=.6778723796
      G(37)=.7240341309  $  G(38)=.7671590325  $  G(39)=.8070662040
      G(40)=.8435882616  $  G(41)=.8765720203  $  G(42)=.9058791367
      G(43)=.9313866907  $  G(44)=.9529877032  $  G(45)=.9705915925
      G(46)=.9841245837  $  G(47)=.9935301723  $  G(48)=.9987710073
```

```
      W(1)=.003153346052  $  W(2)=.007327553901  $  W(3)=.01147723458
      W(4)=.01557931572  $  W(5)=.01961616046  $  W(6)=.02357076084
      W(7)=.02742650971  $  W(8)=.03116722783  $  W(9)=.03477722256
      W(10)=.03824135107  $  W(11)=.04154508294  $  W(12)=.04467456086
      W(13)=.04761665849  $  W(14)=.05035903555  $  W(15)=.05289018949
      W(16)=.05519950370  $  W(17)=.05727729210  $  W(18)=.05911483970
      W(19)=.06070443917  $  W(20)=.06203942316  $  W(21)=.06311419229
      W(22)=.06392423858  $  W(23)=.06446616444  $  W(24)=.06473769681
      W(25)=.06473769681  $  W(26)=.06446616444  $  W(27)=.06392423858
      W(28)=.06311419229  $  W(29)=.06203942316  $  W(30)=.06070443917
      W(31)=.05911483970  $  W(32)=.05727729210  $  W(33)=.05519950370
      W(34)=.05289018949  $  W(35)=.05035903555  $  W(36)=.04761665849
      W(37)=.04467456086  $  W(38)=.04154508294  $  W(39)=.03824135107
      W(40)=.03477722256  $  W(41)=.03116722783  $  W(42)=.02742650971
      W(43)=.02357076084  $  W(44)=.01961616046  $  W(45)=.01557931572
      W(46)=.01147723458  $  W(47)=.007327553901  $  W(48)=.003153346052
      A1=6.36739E6
C
C                    INPUT DATA

   10 READ 1,CNDK,EPS2,DMIN,DELTA,DMAX,NHOP,NT,POWER,KASE,ICOND,
     1(XD(I),NH(I),I=1,ICOND)

C         CNDK = GROUND CONDUCTIVITY IN MHOS/METER
C         EPS2 = DIELECTRIC CONSTANT OF THE GROUND RELATIVE TO FREE SPACE
C         NOTE.  THE GROUND WAVE AND THE FIRST NHOP HOPS WILL BE
C         CALCULATED EVERY DELTA KM. FROM DMIN KM. TO DMAX KM USING NHOP  HOPS,
C         THEN ON TO XD(1) KM USING NH(1) HOPS,
C         THEN ON TO XD(2) KM USING NH(2) HOPS,
C         AND SO ON TO ICOND SECTIONS.  THE MAX OF NHOP OR NH IS 9.
C         NT = NUMBER OF ANGLES OF INCIDENCE FOR WHICH THE REFLECTION
C         COEFFICIENTS ARE READ INTO THE PROGRAM
C         POWER IS THE RADIATED POWER IN WATTS RELATIVE TO DIPOLE IN HALF SPACE
C         KASE = 1, THE AMPLITUDE AND PHASE OF THE WAVE HOPS ARE
C         PUNCHED ON DATA CARDS - KASE = 0, NO DATA CARDS ARE OBTAINED

      DS=-1.
      IF = 1

C         CONTROL VARIABLES.  DS IS DISTANCE MEMORY IN INTEGRATION LOOP.
C         IF IS USED TO INDICATE FIRST OR SUBSEQUENT ENTRY TO CWGW.
C         DSAV AND KMAX ARE USED IN LPAINR TO PREVENT RECALCULATION OF
C              AVAILABLE NUMBERS.

      DSAV=-1.   $   KMAX=0
      ICD=0
      MHOP=NHOP
      XDIST=DMAX
      IF(EOF,60)999,20
   20 IF(ICOND .LE. 0) GO TO 25
      MHOP=NH(ICOND)
      XDIST=XD(ICOND)
C
C                    IDENTIFICATION OF THE PROFILE
```

```
   25 READ 4,ID(1),ID(2),ID(3),ITIM,IPHI,PHIA,ID(4),ID(5),ID(6),ID(7),
     7ID(8)

C           THESE PARAMETERS ARE FOR IDENTIFICATION ONLY AND USUALLY
C                INCLUDE A NAME, TIME, GEOGRAPHIC AZIMUTH, DIP ANGLE,
C                AND MAGNETIC FIELD INTENSITY.
C
C                     READ ANISOTROPIC REFLECTION COEFFICIENTS

      DO 30 N=1,NT
      READ 3,FREQ,PHI(N),(TAMP(L,N),TPHA(L,N),L=1,4),HBOT,HREF(N)

C           FREQ = FREQUENCY IN KHZ
C           PHI = ANGLE OF INCIDENCE IN DEGREES
C           TAMP = AMPLITUDE OF REFLECTION COEFFICIENT
C           TPHA = PHASE OF REFLECTION COEFFICIENT
C           NOTE.  THE REFLECTION  COEFFICIENTS SHOULD BE GIVEN IN THE
C           FOLLOWING ORDER- TEE, TEM, TME AND TMM
C           HBOT = BOTTOM OF THE PROFILE IN KM
C           HREF = REFLECTION HEIGHT IN KM

   30 CPHI(N)=COSF(PHI(N)*.01745329252)
      OMEGA=FREQ*6283.185307
      QKA1=WAVE=OMEGA/2.997925E8
      NPI=(CPHI(NT-1)*TPHA(1,NT)-CPHI(NT)*TPHA(1,NT-1))/
     1(6.283185307*(CPHI(NT-1)-CPHI(NT)))
      H=HREF(NT)+NPI*6.283185307/(2.*WAVE*CPHI(NT))*.001
C
C                     CALCULATION OF THE VARIABLES THAT ARE NOT A FUNCTION
C                     OF DISTANCE

      AK1=A1*QKA1
      A2=A1+H*1.E3
      SQ2=CMPLX(WAVE*WAVE*EPS2,-12.56637061E-7*OMEGA*CNDK)
      QKA2=CSQRT(SQ2)
      R12=QKA1/QKA2
      R21=QKA2/QKA1
      V=CUBERTF(AK1/2.)
      V2=V*V
      Y=QKA1*H*1000./V
   33 FK= 30.* SQRT(3.141592653*POWER/120.)
      Z=1.25/V2
      Q(1)=V*R12*(0.,-1.)*CSQRT(1.-R12*R12)
      ETA=R21*R21
      Q(2)=ETA*Q(1)
C
C                     INITIALIZE
C                     NN=1     SIGNALS FIRST TIME IN NUMERICAL INTEGRATION

      NN=1
      DO 35  J=1,MHOP
      AMP(J)=0.
   35 PH(J)=0.
```

```
C                   PRINT LABELS AND VALUES OF CONSTANTS

      PRINT 5,POWER,FREQ,CNDK,EPS2
      PRINT2,ID(1),ID(2),ID(3),ITIM,IPHI,PHIA,ID(4),ID(5),ID(6),ID(7),
     7ID(8),H
      PRINT 36
   36 FORMAT( *0   PHI  COS(PHI)*, 7X, *T E E*, 15X, *T E M*, 15X, *T M
     1E*,15X, *T M M*, 12X, *HBOT*, 5X, *HREF*/
     215X, 4(*   AMPLITUDE  PHASE *))
C
C                   ADJUST PHASE OF T TO REFERENCE HEIGHT
      DO 55 L=1,4
      DO 40 N=1,NT
      TPHA(L,N)=TPHA(L,N)+2.*WAVE*(H-HBOT)*CPHI(N)*1000.
      IF(TPHA(L,N).GT. 6.283185307) TPHA(L,N)=MODF(TPHA(L,N),
     1 6.283185307)
   40 CONTINUE

C         MAKE PHASE OF REFLECTION COEFFICIENT CONTINUOUS AS A FUNCTION
C              OF  COS(PHI)

      CALL GUDFAZ(L,CPHI,TPHA,NT)
C
C                   COMPUTE A FOR THIS ANGLE

      DO 50 N=1,NT
      IF( L .EQ. 2 .OR. L .EQ. 3) GO TO 47
      IF ( L .EQ. 4)
     1PRINT 37, PHI(N), CPHI(N),(TAMP(I,N),TPHA(I,N),I=1,4),HBOT,HREF(N)
   37 FORMAT( F7.2, F8.4, 4(F10.5, F10.3), 2F10.1)
   45 TPR(L,N)=LOGF(TAMP(L,N))/CPHI(N)
      TPI(L,N)=(TPHA(L,N)-3.141592653)/CPHI(N)
      GO TO 50
   47 TPR(L,N) = TAMP(L,N)
      TPI(L,N) = TPHA(L,N)
   50 CONTINUE
   55 CONTINUE
C
      PRINT 39
   39 FORMAT(*0DISTANCE*,8X,*TOTAL*, 5X, *GR WAVE*, 3X, *HOP 1*, 5X,
     1*HOP 2*, 5X, *HOP 3*, 5X, *HOP 4*, 5X, *HOP 5*, 5X, *HOP 6*, 5X,
     2*HOP 7*, 5X, *HOP 8*, 5X, *HOP 9*)
C
C
C     INITIALIZE JGO,     METHOD SELECTOR FLAG FOR CALCULATING I SUB J,M
C
C                   GEOMETRICAL OPTICS FOR SHORTEST DISTANCES    JGO=4
C                   SADDLE POINT FOR SHORT DISTANCES             JGO=3
C                   NUMERICAL INTEGRATION FOR LONG DISTANCES     JGO=2
C                   RESIDUE SERIES FOR LONGEST DISTANCES         JGO=1
C                       (UNLESS NHOP .GT. 5, THEN USE NUM. INTEG.)

      DO 79  J=1,MHOP
```

```
      JGO(J)=4
      IF(COSF(DMIN/(.002*J*A1))  .LE.  (A1/A2))   JGO(J)=1
   79 CONTINUE
C
C                    BEGINNING OF THE LOOP DMIN (DELTA) DMAX  - LOOP ENDS
C                    AT 995

   80 THETA=DMIN*1.E3/A1
      X=V*THETA
      X2=X*X
      STH=SIN(THETA)
      AK1D=QKA1*DMIN*1000.
      F3=FK/SQRT(A1*A1*THETA*STH)
      F2= SQRT(2.*X/3.141592653)*F3*(-1.,1.)
      F1= (0., 6.283185307) *F2
      DO 85  J=1,MHOP
      DO 85 M=1,J
   85 HOP(J,M)=0
C
C                    CALCULATION OF E SUB 0, THE GROUND WAVE

      CALL CWGW( IF, DMIN, 0.,0.,1., GW , Q(1), X,Z)
      SUM = GW
C
C        BEGIN HOP LOOP
C
      DO 980  J=1,NHOP
      TUJ=2*J
      TOTJ=THETA/TUJ
      GO TO (700,500,300,100),JGO(J)
C
C
C          BEGIN GEOMETRICAL OPTICS METHOD OF CALCULATING I SUB J,M
C
  100 DJ=TUJ *SQRTF(2.*A1*A2*OMCOS(TOTJ)+H*H*1.E6)
      SINTAU=TUJ *A2*SINF(TOTJ)/DJ
      COSSQT=1.-SINTAU*SINTAU
      COSTAU=SQRTF(COSSQT)
      COSPHI =(DJ+TUJ*A1*COSTAU)/(TUJ*A2)
      COS5=COSSQT*COSSQT*COSTAU

C        IS COS TAU SMALL ENOUGH TO USE SADDLEPOINT FORMULA

      IF(COS5  .LE.  1./AK1)      GO TO 295
  150 BJ=A2/A1*SQRTF(DJ*SINTAU*COSPHI/(A2*STH*COSTAU))
      S=(0.,1.)*V*COSTAU
      C1 = CMPLX(COSPHI,0.)
      RM=(S+Q(2))/(S-Q(2))
      RE=(S+Q(1))/(S-Q(1))
      P= QKA1*DJ-AK1D
      S1=BJ*SINTAU*SINTAU* FK/DJ*(1.+RE)**2
     1 * CMPLX(COS(P),-SIN(P))
      HOP(J,1) = S1*RE**(J-1)
      DO 175 M=2,J
```

```
  175 HOP(J,M) = HOP(J,M-1)*RM/RE
      GO TO 900
C
  295 JGO(J)=JGO(J)-1
C
C
C          BEGIN SADDLE POINT METHOD OF CALCULATING I SUB J,M
C
  300 P=(4.*J*J*Y-X2)/(4.*X*J)

C        IS P SO SMALL WE MUST INTEGRATE NUMERICALLY

      IF(P  .LT.  2.+(J-1.)*.15)     GO TO 495
  301 P2=P*P
      DJ=SQRT(Y+P2)
      C1= CMPLX (DJ/V,0.)
      OMEGA = -X*P2+1.333333*J*((Y+P2)*DJ-P2*P)
      S2= CMPLX(0.,P)
      RE=(S2+Q(1))/(S2-Q(1))
      RM=(S2+Q(2))/(S2-Q(2))
      S = SQRT((1.+X/(2*J*P))*(1.-.5*P2/V2)**5)*(1.+RE)**2
     1 *CMPLX(COS(OMEGA), -SIN(OMEGA))*RE**(J-1)
      HOP(J,1) = F3*S
      DO 475 M=2,J
  475 HOP(J,M) = HOP(J,M-1)*RM/RE
      GO TO 900
C
  495 JGO(J)=JGO(J)-1
C
C
C                   THE PATH INTEGRALS ARE CALCULATED USING NUMERICAL
C                   INTEGRATION.                     GAUSSIAN INTEGRATION
C                   IS USED WITH 48 ABSCISSAS AND WEIGHTS.  THE CONTOURS
C                   OF INTEGRATION CONSIST OF TWO SEGMENTS,  T GOING
C                   FROM 4 TO 0 AND THEN INTO THE LOWER HALF PLANE AT A
C                   SLOPE OF 1/4
C
  500 IF(COSF(THETA/TUJ)          .LE.  A1/A2)     GO TO 695

C        IF WE ARE IN THE SHADOW REGION (TAU .GE. 90 DEGREES)
C             USE RESIDUE SERIES.

  501 IF(NN.EQ.2)GO TO 600
C
C                   THE FIRST TIME A HOP IS CALCULATED WITH NUMERICAL
C                   INTEGRATION THE COEFFICIENT OF E**(-IXT) IS
C                   DETERMINED FOR THE GAUSSIAN ABSCISSAS AND STORED FOR
C                   FUTURE USE
      TOP=0.
      BOT=4.
      KK=0
      DO 590 L=1,2
      DO 575 K=1,48
      KK=KK+1
```

```
      IF(L.EQ.2)GO TO 520
      OI=0.
      OR=((TOP-BOT)*G(K)+TOP+BOT)*.5
      GO TO 525
  520 OI=((TOP-BOT)*G(K)+TOP+BOT)*.5
      OR= OI*4.
  525 O(KK)=CMPLX(OR,OI)
      CALL CWAIRY(1,O(KK),T1,M1,T2,M2)
      CALL CWAIRY(2,O(KK),DT1,MD1,DT2,MD2)
      CALL CWAIRY(1,O(KK)-Y,TY1,MTY1,TY2,MTY2)
      S2=DT1*(2.718281828**MD1)-Q(1)*T1*(2.718281828**M1)
      R1=(1.+Z*O(KK))/(S2*S2)
      R2=2.718281828**(MTY1-MTY2)*TY1/TY2
      R3=2.718281828**(M2-M1)*T2/T1
      S1=2.718281828**(MD2-M2)*DT2/T2
      S2=2.718281828**(MD1-M1)*DT1/T1
      R4=(S1-Q(1))/(S2-Q(1))
      R5=(S1-Q(2))/(S2-Q(2))
      S1= (1.,0.)
      RR= R1/R3
      JM=0
      DO 575  JJ=1,MHOP
      RR =-RR*R2*R3
      JM=JM+1
      PT(JM,KK)=RR*S1
C
C                   PT(JM,KK) = THE ARRAY OF COEFFICIENTS OF E**(-IXT)
C                   FOR THE 48 GAUSSIAN ABSCISSAS
C
C            NOW CALCULATE PT  FOR ALL HOPS
C
      S1= S1*R4
      DO 574 M=2,JJ
      MJ=JM
      JM=JM+1
  574 PT(JM,KK)=PT(MJ,KK)*R5/R4
  575 CONTINUE
      FTX(L)=.5*(TOP-BOT)
      BOT=0.
  590 TOP=-9./(2.*SQRTF(4.+Y)-4.)
      NN=2
  600 KK=0
C
C                   CALCULATION OF THE INTEGRAL

      DO 650 L=1,2
      DO 650 K=1,48
      HH=W(K)*FTX(L)
      KK=KK+1
      IF(DS  .EQ.  DMIN)   GO TO 625
      IF(L.EQ.2)GO TO 620
      ARG=X*REAL(O(KK))
      SD(KK)=CMPLX(COSF(ARG),-SINF(ARG))
      GO TO 625
```

```
  620 SD(KK)=CEXP(CMPLX(0.,-X)*O(KK))
  625 CONTINUE
      JM= (J*(J-1))/2
      DO 650 M=1,J
      JM=JM+1
      IF(L.EQ.2)GO TO 630
C
C                  SUMMING OF THE INTEGRAND FOR THE FIRST INTERVAL
C                                      REAL PLANE

      HOP(J,M)=HOP(J,M)+HH*PT (JM,KK)*SD(KK)*F2
      GO TO 650
C
C                  SUMMING OF THE INTEGRAND FOR THE SECOND INTERVAL
C                                      COMPLEX PLANE

  630 B=PT (JM,KK)*SD(KK)
      S1=B*(4.,1.)
      HOP(J,M) = HOP(J,M) +HH*S1*F2
  650 CONTINUE
      ALP=(4.*J*J*Y-X2)/(4.*X*J)
      C1= CMPLX(SQRT(Y+ALP*ALP)/V,0.)
      DS=DMIN
      GO TO 900
C
  695 IF(J  .GT.  5)     GO TO 501
      JGO(J)=JGO(J)-1
C
C
C                  THE PATH INTEGRALS ARE CALCULATED WITH THE RESIDUE
C                  SERIES
C
  700 MO=0
  800 DO 850 K=1,20

C        LPAINR COMPUTES THE RESIDUES FOR ALL M FOR THIS J.

      CALL LPAINR(J,K,R)
      DO 825 M=1,J
      HOP(J,M) = HOP(J,M)-(R(1,J,M)+R(2,J,M))*F1
      IF(K.EQ.1)GO TO 825
      IF(CABS(((R(1,J,M)+R(2,J,M))*F1)/HOP(J,M))  .GT. 0.0005) GO TO 825
      MO=MO+1
      IF(MO .NE.  J)    GO TO 825
      GO TO 885
  825 CONTINUE
  850 CONTINUE
  885 CONTINUE
      DCRIT = SQRT( .008*A1*H)
      S1= MIN1F( 1., (DMIN-J*DCRIT)/(300*J))*T(1)
      C1= CSQRT((2000.*H/A1-S1    /V2)/(1.+2000.*H/A1))
C
C                  COMPUTE TEE,TEM,TME AND TMM
```

```
  900 CONTINUE
      IF(JGO(J) .EQ. 1) GO TO 904
  895 C1=  CMPLX( (A1*OMCOS(TOTJ)+H*1000.)/SQRT(2.*A1*A2*OMCOS(TOTJ) +
     1 H*H*1.E6),0.)
C
C        IF PHI IS MUCH SMALLER THAN ANY INPUT VALUE, SET TA (AND HENCE
C             THIS HOP) TO A VERY SMALL VALUE AS AN INDICATOR.
C
  904 CONTINUE
      IF(REAL(C1)  .LT.  1.2*CPHI(1))  GO TO 905
      TA(J,1)=TA(J,2)=TA(J,3)=TA(J,4)=1.E-25
      GO TO 940
  905 C2=C1*C1
      NL=NT-1
      NI=1     $ NIP = NI+1
      CY = REAL(C1)
C
C        DETERMINE WHICH VALUES OF  T(COS(PHI))  TO USE IN INTERPOLATION
C
      IF( CY  .GT. CPHI(1)) GO TO 908
  906 IF((CY .LE. CPHI(NI)) .AND. (CY .GT. CPHI( NIP))) GO TO 908
      IF(NI .GE. NL)  GO TO 908
      NI = NIP     $ NIP = NIP +1
      GO TO 906
  908 CX = CPHI (NI) - CPHI(NIP)
C
C        INTERPOLATE TO GET  A SUB 1  AND  A SUB 2
C
      DO 925 L=1,4
      ATR(  2) = ( TPR(L,NI) - TPR( L, NIP))/CX
      ATI(  2) = ( TPI(L,NI) - TPI( L, NIP))/CX
      ATR(   1) = TPR(L,NIP) -ATR(  2)*CPHI(NIP)
      ATI(   1) = TPI(L,NIP) -ATI(  2)*CPHI(NIP)
      IF(L.EQ.1.OR.L.EQ.4)GO TO 920
      DY=ATI(  1)+ATI(  2)*REAL(C1)
      DX=EXP(-ATI(  2)*AIMAG(C1))
      TA(J,L)=(ATR(  1)+ATR(  2)*C1)*DX*CMPLX(COS(DY),SIN(DY))
      GO TO 925
  920 S=CMPLX(ATR(  1),ATI(  1))*C1+CMPLX(ATR(  2),ATI(  2))*C2
      TA(J,L) = -CEXP(S)
  925 CONTINUE
C
C                  CALCULATION OF C SUB J,M
C
  940 E(J)=HOP(J,1)*TA(J,1)**J
      DO 960 M=2,J
      C=0.
      AJMK=J-M+1
      M1=M-1
      DO 950 K=1,M1
      IF(K.EQ.1)GO TO 950
      AJMK=(J+2-M-K)*(M-K)/(K*(K-1))*AJMK
  950 C      =C      +AJMK*TA(J,1)**(J+1-M-K)*(TA(J,2)*TA(J,3))**K*TA(J,4)
     1**(M-1-K)
```

```
C
C                     CALCULATION OF E SUB J, THE WAVE HOPS.

  960 E(J)=E(J)+HOP(J,M)*C

C                     CALCULATION OF E SUB R, THE VERTICAL ELECTRIC FIELD

      SUM=SUM+E(J)
      AMP(J)=CABS(E(J))
  980 PH(J) = -CANG(E(J))
C
C      END OF J LOOP
C
      GAMP=CABS(GW)
      GPH = -CANG(GW)
      SAMP=CABS(SUM)
      SPH = - CANG(SUM)
C
C                     PRINT AND PUNCH OUTPUT
C
      PRINT 8, DMIN, SAMP, GAMP, (AMP(J), J=1,NHOP)
      PRINT 9, SPH, GPH, (PH(J), J=1,NHOP)
  990 IF(KASE.EQ.0)GO TO 995
      PUNCH 12,FREQ,ID(1),ID(2),ITIM,PHIA,DMIN,SAMP,CNDK
   12 FORMAT(F7.1,2A8,2X,A8,A6,F9.2,E20.8,F6.1)
C
C                     END OF LOOP DMIN (DELTA) DMAX

  995 DMIN=DMIN+DELTA
      IF(DMIN-DMAX)80,80,1000
  999 CALL EXIT
 1000 IF(ICD .GE. ICOND)  GO TO 10
      ICD=ICD+1
      DMAX=XD(ICD)
      NHOP=NH(ICD)
      GO TO 80
      END
```

```
      SUBROUTINE LPAINR(J,K,R)
C     SUBROUTINE FOR ANIHOP.   COMPUTES RESIDUES FOR GIVEN J,K, AND
C             DISTANCES.
C                                  IMPORTANT
C        ARRAYS E AND A MUST HAVE THE SAME FIRST DIMENSION FOR USE
C                                      IN DEPROD
      TYPE INTEGER TUJ
      COMPLEX Q,T,W,TE,E,A,BJM,AJM,SUM,DA,ZX,ARG,EX,AHAT,R
      DIMENSION Q(2),T(40),W(8,2,2),MW(20,2,4),TE(2),E(7,2,2),A(7,15)
     1,BJM(6,20,6),AJM(7,20,15),DA(7),EX(6,40),AHAT(6),R(2,5,5)
      COMMON/PAIN/X,Y,Z,DMIN,XDIST,DSAV,A1,A2,Q,KMAX,T
      COMMON/PROD/C
      COMPLEX  C(7,15)
      DATA (EC=2.718281828)
      NP(N) = IZ*5+IS*N
      LP(L,N)= L+IZ*(IZ*5+IS*N)
C        IF THIS IS THE FIRST ENTRY FOR THIS MODEL, COMPUTE JRES
      IF(DSAV  .GT.  0.)   GO TO 5
      JRES=MINOF(5,XFIXF(XDIST/(.002*A1*ACOSF(A1/A2))))
      KEX=0
    5 IQ=1
C        IQ=1 FOR Q SUB E POLES.  IQ=2 FOR Q SUB M POLES.
      IZ=0
      IS=1
      NDER=JRES+1
      KK=K   $   JN=J+1
      IW=1   $   IY=2
      SIGN=(-1)**(J-1)
C        IF WE HAVE COMPUTED A S  AND B S  FOR THIS K, GO TO 240
   10 IF(K  .LE.  KMAX)   GO TO 240
C        FROM HERE TO 200,  COMPUTE  AJM  AND BJM  FOR THIS K.
C        FIRST, FIND T(K) AND AIRY FUNCTIONS OF T.
C        TW FINDS SOLUTIONS OF E1(T,Q)=0.
   20 CALL TW(K-1,Q(IQ),T(KK),W(1,1,1),MW(K,1,IW),W(2,1,1),MD1,
     AW(1,2,1),MW(K,2,IW),W(2,2,1),MD2)
      TE(1)=T(KK)
      TE(2)=TE(1)-Y
      W(2,1,1)=(EC**(MD1-MW(K,1,IW)))*W(2,1,1)
      W(2,2,1)=(EC**(MD2-MW(K,2,IW)))*W(2,2,1)
      CALL CWAIRY(1,TE(2),W(1,1,2),MW(K,1,IY),W(1,2,2),MW(K,2,IY))
      CALL CWAIRY(2,TE(2),W(2,1,2),MD1,W(2,2,2),MD2)
      W(2,1,2)=(EC**(MD1-MW(K,1,IY)))*W(2,1,2)
```

```
      W(2,2,2)=(EC**(MD2-MW(K,2,IY)))*W(2,2,2)

C         NOW GET HIGHER DERIVATIVES OF W AND E

      NTOP=NDER+2
      DO 30  N=3,NTOP
      DO 29  I=1,2
      DO 28  KIND=1,2
      W(N,KIND,I)=TE(I)*W(N-2,KIND,I)+(N-3)*W(N-3,KIND,I)
      E(N-1,KIND,I)=W(N,KIND,1)-Q(I)*W(N-1,KIND,1)
      IF(N  .GT.  3)    GO TO 28
      E(1,KIND,I)=W(2,KIND,1)-Q(I)*W(1,KIND,1)
   28 CONTINUE
   29 CONTINUE
   30 A(N-2,1)=W(N-2,1,2)

C         W SUB ONE (T-Y)  AND DERIVATIVES ARE IN COL. 1  OF A.

C         NOW COMPUTE BJM

   40 BJM(LP(1,1),K,NP(1))=E(2,1,IQ)
      DO 50  N=2,NDER
      BJM(LP(1,N),K,NP(N)) = BJM(LP(1,N-1),K,NP(N-1))*E(2,1,IQ)
      DO 48  L=2,N
      SUM=0.
      KF=1
      KKT=L-1
      DO 46  KI=1,KKT
      KF=KF*(KI+1)
      SUM=(KI*(N+1)-KKT)/(KKT*KF)*BJM(LP(L-KI,N),K,NP(N)) *
     1 E(KI+2,1,IQ)+SUM
   46 CONTINUE
   48 BJM(LP(L,N), K, NP(N)) = SUM/E(2,1,IQ)
   50 CONTINUE

C         DERINV FINDS DERIVATIVES OF 1/F.

      CALL DERINV(W(1,2,2),NDER-1,A(1,2))

C         GET  W/W  AND DERIVATIVES AND PUT IN COL. 2  OF  A.


C         DEPROD COMPUTES DERIVATIVES OF A PRODUCT OF N FUNCTIONS.

      CALL DEPROD(A,2,NDER-1,A(1,2))

C         NOW PUT  D(2)  AND DERIVATIVES IN  COL. 1 OF  A.

      A(1,1)=1.+Z*TE(1)
      A(2,1)=Z
      A(3,1)=A(4,1)=A(5,1)=A(6,1)=0.

      GO TO (70,150),IQ
```

```
C        AJM FOR QE POLES

   70 CONTINUE
      CALL DERINV(E(1,1,2),NDER-1,E(1,1,2))
      CALL DEPROD(E(1,1,2),2,NDER-1,A(1,15))

C        E2/E1 AND ITS DERIVATIVES ARE NOW IN COL 15 OF A.
C        NOW CALCULATE AJM/EXP(-IXT)

      JRES2 = JRES+JRES
      DO 75 JX=4,JRES2,2
      DO 74 I=1,NDER
      A(I,JX)= E(I,2,1)
   74 A(I,JX-1) = A(I,2)
   75 CONTINUE
      DO 90 M=1,JRES
      CALL DEPROD (A,JRES2,JRES-M+1,DA)
      DO 80 JX=M,JRES
      JM= JX-5+(M*(11-M))/2
      IM= JX-M+2
      DO 78 I=1,IM
   78 AJM(I,K,JM) = C(I,JX+JX)
   80 CONTINUE
      IF(M .EQ. JRES) GO TO  240
      JP = M+M+2
      IM=NDER-M
      DO 82 I=1,IM
   82 A(I,JP) = A(I,15)
   90 CONTINUE
      GO TO 240

C        AJM  FOR  QM POLE

  150 AJM(3,K,6) = A(1,1)*E(1,2,2)*(A(1,2)/E(1,1,1))**2
      IF(JRES  .LT.  3)   GO TO 200
      ZX=A(1,2)*E(1,2,1)/E(1,1,1)
      CALL DERINV(E(1,1,1), NDER-1, E(1,1,1))
      DO 160  N=1,NDER
      A(N,4)=A(N,3)=A(N,2)
      A(N,6)=A(N,5)=E(N,1,1)
      A(N,8)=A(N,7)=E(N,2,2)

      JX=N+2
      IF(N  .GE.  NDER)   GO TO 161

C             STATEMENT 160 GETS AJM SUB ZERO FOR J=N+1, M=2

  160 AJM(1+JX,  K, N+6) = AJM(JX,K,N+ 5) *ZX
  161 CONTINUE

C             GET AJM SUB ZERO AND ONE  FOR  J=3, M=3

      CALL DEPROD(A,8,NDER-1,A(1,1))
      AJM(3,K ,10)=A(1,1)
```

```
      AJM(4, K,10)=A(2,1)
      IF( JRES  .LT.  4)   GO TO 200
      CALL DEPROD(E(1,1,1),2,NDER-1, A(1,5))
      DO 170  N=1,NDER
      A(N,3)=A(N,5)
  170 A(N,4)=A(N,2)
      DO 190  M=3,JRES
      JM=(M*(11-M))/2-1
      M1=M-1
  175 CALL DEPROD(A,5,M-2,AJM(8-M,K,JM+1))
      IF(M  .EQ.  5)   GO TO 200
      DO 176 I=1,M1
  176 AJM(I+6-M,K,JM) = C(I,3)
  178 MF=2*M-3
      DO 180  N=1,NDER
  180 A(N,MF)=E(N,2,2)
  190 CONTINUE
  200 CONTINUE
      KMAX=K
  240 IF(DSAV  .NE. DMIN)   GO TO 250
      IF(K  .LE.  KEX)   GO TO 300

C         COMPUTE EXP(-IXT) AND DERIVATIVES.

  250 ARG=CMPLX(0.,-X)
      EX(1,KK)=CEXP(ARG*T(KK))
      DO 260  N=2,NDER
  260 EX(N,KK)=ARG*EX(N-1,KK)
      IF ( IQ .EQ. 2) KEX=K
  300 SCALE=2.718281828**(J*(MW(K,1,IY)-MW(K,2,IY))+(J-1)*(MW(K,2,IW)
     A-MW(K,1,IW)))*SIGN

C         COMPUTE RESIDUES FOR I-SUB-JM FOR THIS POLE.

      DO 330  M=1,J
      JB=J-M+2
      IN=0
      IF(IQ .EQ. 1) GO TO 301
      IN=JB
      JB=M-1
  301 JM=J-5+(M*(11-M))/2
      IF(JB-1)302,303,304
  302 R(IQ,J,M)=0.
      GO TO 330
  303 R(IQ,J,M) = SCALE*AJM(1+IN, K, JM) *EX(1,KK)/BJM(LP(1,1),K,NP(1))
      GO TO 330
  304 DO 305  N=1,JB
      A(N,1) = AJM(N+IN, K,JM)
  305 A(N,2)=EX(N,KK)

C         COMPUTES A-SUB-N,  EQ(36).

      CALL DEPROD(A,2,JB-1,DA)
      NPB = NP(JB)
```

```
      FACT=1.
C           NOW COMPUTE A-HATS USING (38).
      AHAT(1)=DA(1)/BJM(LP(1,JB),K,NPB)
      DO 315  N=2,JB
      DA(N)=DA(N)/FACT
      SUM=DA(N)
      IM=N-1
      DO 312  L=1,IM
  312 SUM=SUM-BJM(LP(N-L+1,JB),K,NPB       )*AHAT(L)
      AHAT(N)=SUM/BJM(LP(1,JB),K,NPB)
  315 FACT=FACT*N
      R(IQ,J,M)=SCALE*AHAT(JB)
  330 CONTINUE
      IF(IQ  .EQ.  2)  GO TO 350
      IQ=2
      IZ=1
      IS=-1
      KK=K+20
      IW=3
      IY=4
      JN=J-1
      NDER=JRES-1
      GO TO 10
  350 DSAV=DMIN
      RETURN
      END
```

```
      SUBROUTINE DEPROD(A,JF,JN,DA)
C     INPUT
C     A(I,J)=THE (I-1)TH DERIVATIVE (I=1,JN+1) OF FUNCTION A(I,J),(J=1,JF)
C     OUTPUT
C     DA(I)=THE (I-1)TH DERIVATIVE OF THE PRODUCT A(1,1)*A(1,2)*...A(1,JF)
C                                IMPORTANT
C        INPUT AND OUTPUT ARRAYS IN DEPROD MUST HAVE SAME FIRST
C                    DIMENSION FOR USE WITH LPAINR
C        MAKES REPEATED USE OF LEIBNITZS RULE.
      COMMON/PROD/C
      COMPLEX A(7,15),DA(15),C(7,15)
      N1=JN+1
      DO 25 N=1,N1
   25 C(N,1)=A(N,1)
      DO 50 M=2,JF
   50 C(1,M)=C(1,M-1)*A(1,M)
      DO 75 M=2,JF
      DO 74 N=2,N1
      C(N,M)=C(1,M-1)*A(N,M)
      FACN=1.
      FACL=1.
      DO 73 L=2,N
      FACN=FACN*(N-L+1)
      FACL=FACL*(L-1)
   73 C(N,M)=C(N,M)+FACN/FACL*C(L,M-1)*A(N-L+1,M)
C        C(N,M)=N TH DERIVATIVE OF PRODUCT (A(1,L),L=1,M)
   74 CONTINUE
   75 CONTINUE
      DO 100 I=1,N1
  100 DA(I)=C(I,JF)
      RETURN
      END
```

```
      SUBROUTINE CWGW ( MM, DMIN, H1, H2, ALFA, GW , Q, X,Z)

C     CALCULATION OF THE GROUND WAVE
C                    INPUT
C                    DMIN = DISTANCE BETWEEN TRANSMITTER AND RECEIVER IN KM
C                    H1 = HEIGHT OF THE TRANSMITTER IN KM
C                    H2 = HEIGHT OF THE RECEIVER IN KM
C                    ALFA = EFFECTIVE EARTH RADIUS
C                    Q = -I*V*(NORMALIZED SURFACE IMPEDANCE)
C                    X = V*THETA
C                    Z = 1.25/V**2

C                    OUTPUT
C                    GW = THE GROUND WAVE

      COMMON/WGW/QKA1,QKA2,FK,AK1,V,V2, THETA,STH
      COMPLEX T(200),W(200),G,QKA2,R12,S,D,Q,W1,DW1,WY1,WY2,GW,S2,TS
      Y1=QKA1*H1*1000./V
      Y2=QKA1*H2*1000./V
      COTH=COS(THETA)/STH
      FAC=FK*SQRT(6.283185307 *X/((6.36739E6*ALFA)**2*THETA*STH))
      GW=0.


C         IF THIS IS THE FIRST ENTRY COMPUTE T-SUB-S AND
C                COEFFICIENT OF EXP(-IXT)
      GO TO(60,125),MM
   60 J2=1
      MM=2
   65 DO 100 J=J2,200

C         FIND LOCATION OF POLE.

      CALL TW( J-1 , Q, T(J), W1, MW1, DW1, MD1, S,M,S,M)
      IF(H1.GT.0)GO TO 80
      IF(H2.GT.0)GO TO 75
      W(J)=1.
      GO TO 85

C         HEIGHT GAIN FUNCTIONS.

   75 CALL CWAIRY(1,T(J)-Y2,WY2,MY2,S,M)
      W(J)=2.7182818**(MY2-MW1)*WY2/W1
      GO TO 85
   80 CALL CWAIRY(1,T(J)-Y1,WY1,MY1,S,M)
      W(J)=2.7182818**(MY1-MW1)*WY1/W1
      IF(H2.GT.0)GO TO 85
      CALL CWAIRY(1,T(J)-Y2,WY2,MY2,S,M)
      S=2.7182818**(MY2-MW1)*WY2/W1
      W(J)=W(J)*S
   85 W(J)=W(J)*(1.+Z*T(J))/(T(J)-Q*Q)

C         COMPUTE TERM OF RESIDUE SERIES AND ADD.
```

```
      G=(1.+(CMPLX(3.,COTH))/(8.*(AK1+V*T(J))))*CEXP((0.,-1.)*X*T(J))
      G=W(J)*G
      GW=GW+G
      IF(J.EQ.1)GO TO 100
      IF( CABS(G/GW)  .GT. 0.0005) GO TO 100
      J1=J
      GO TO 110
  100 CONTINUE
      J2=200
      GO TO 165
  110 IF(J1.LE.J2)GO TO 165
      J2=J1
      GO TO 165
  125 DO 140 J=1,J2
      G=(1.+(CMPLX(3.,COTH))/(8.*(AK1+V*T(J))))*CEXP((0.,-1.)*X*T(J))
      G=W(J)*G
      GW=GW+G
      IF(J.EQ.1)GO TO 140
      IF( CABS(G/GW)  .LT. 0.0005) GO TO 165
  140 CONTINUE
      IF(J2.GE.200)GO TO 165
      J2=J2+1
      GO TO 65
  165 GW=GW*FAC *(1.,-1.)
      RETURN
      END
```

```
      SUBROUTINE ZEXP(A,B,X,Y,MAGTUD)
C     SCALED EVALUATION OF THE EXPONENTIAL FUNCTION IN THE COMPLEX PLANE.
C     INPUT.
C       (A+IB) = THE COMPLEX EXPONENT.

C     OUTPUT.
C       EXP(A+IB) = (X+IY)*(E**MAGTUD)

      MAGTUD=A
      SCALE=MAGTUD
      E=EXPF(A-SCALE)
      X=E*COSF(B)
      Y=E*SINF(B)
      RETURN
      END
```

```
      FUNCTION OMCOS(X)
C     OMCOS(X)= 1 - COS(X)
C      IS ACCURATE FOR ALL X INCLUDING X NEAR 0.
      IF(ABSF(X).GT..15) GO TO 40
      IF(X.EQ.0.) GO TO 50
C        IF X IS SMALL, SUM TAYLORS SERIES FOR 1-COS(X)
      S = X*X
      OMCOS = T = .5*S
      R = 4.
   10 T=-T*S/(R*(R-1.))
      OMCOS=OMCOS + T
      IF(ABS(T/OMCOS) .LE. .5E-9) GO TO 51
      R=R+2.
      GO TO 10
   40 OMCOS = 1.-COS(X)
      RETURN
   50 OMCOS = 0.
   51 RETURN
      END
```

```
      SUBROUTINE DERINV(F,N,EF)
C     INPUT
C     F(K) = THE (K-1)TH DERIVATIVE OF F, K=1,N+1
C     OUTPUT
C     EF(K) = THE (K-1)TH DERIVATIVE OF 1/F,K=1,N+1
      COMPLEX F(15),DF(15),EF(15)
      DF(1)=1./F(1)
      DO 50 K=1,N
      K1=K+1
      FACK=1.
      FACM=1.
      DF(K1)=0.
      DO 50 M=1,K
      FACK=FACK*(K1-M)
      FACM=FACM*M
   50 DF(K1)=DF(K1)-(FACK/FACM*F(M+1)*DF(K1-M))/F(1)
      M=N+1
      DO 60  K=1,M
   60 EF(K)=DF(K)
      RETURN
      END
```

```
      SUBROUTINE TW( I, Q, T, W1,MW1, DW1,MD1, W2,MW2, DW2,MD2)
C
C  T IS THE I-TH ROOT OF W-SUB-ONE-PRIME - Q*W-SUB-1 =0.(W IS THE AIRY FUNCTION)
C  THE ROOTS ARE COUNTED IN ORDER OF INCREASING MAGNITUDE.
C  W-SUB-ONE(T) = EXP(MW1)*W1, W-SUB-ONE-PRIME = EXP(MD1)*DW1,
C      W-SUB-TWO = EXP(MW2)*W2, ETC.
C
      DIMENSION TZERO(11), TINFIN(11)
      COMPLEX Q,W1,DW1,W2,DW2, PH, A ,T

C      W-SUB-ONE-PRIME(TZERO(I)) =0.

      DATA (TZERO= 1.018793, 3.2481975, 4.8200992, 6.1633074,
     1 7.3721773, 8.4884868, 9.5354490, 10.52766, 11.475057, 12.384788,
     2 13.262219)

C      W-SUB-ONE(TINFIN(I)) =0.

      DATA (TINFIN = 2.3380997, 4.0879494, 5.5205598, 6.7867081,
     1 7.9441336, 9.0226508, 10.040174, 11.008524, 11.936016, 12.828777,
     2 13.691489)
      DATA (PH =(0.5, -0.8660254)), ( CON= 1.17809724)
C
      IF( REAL(Q)**2 + AIMAG(Q)**2 .GT. 1.) GO TO 50
      IF(I .GT. 10) GO TO 10
      TZ = TZERO(I+1)
      GO TO 20
   10 YS = ((4*I+1)*CON)**2
      TZ = CUBERTF(YS) *(1.-.1458333/YS)
   20 T = TZ*PH

C     T IS NOW SOLUTION FOR Q =0.  THE NEXT STEP IS THE FIRST NEWTON ITERATION.

      T = T+Q/T
      GO TO 100
   50 IF(I .GT. 10) GO TO 60
      TZ = TINFIN(I+1)
      GO TO 70
   60 YS = ((4*I+3) *CON)**2
      TZ= CUBERTF(YS)*(1.+.1041667/YS)
   70 T = TZ*PH

C  T IS SOLUTION FOR Q=INFINITY.  NEXT STEP IS THE FIRST NEWTON ITERATION.

      T = T+1./Q
  100 K=0

C      NOW, USE NEWTONS ITERATION TO CONVERGE ON SOLUTION
C        CWAIRY COMPUTES  W(T)  AND  W PRIME (T)

  101 CALL CWAIRY(1,T,W1,MW1,W2, MW2)
      CALL CWAIRY(2,T,DW1,MD1,DW2,MD2)
      A=(2.718281828**(MD1-MW1))*DW1/W1
      A = (A-Q)/(  T -A*Q)
```

```
      T = T-A
      K=K+1
      IF(K .GT. 15) GO TO 150
      IF(CABS(A/T) .GT. 0.5E-6) GO TO 101
      RETURN
  150 PRINT 155 , I, T, A
  155 FORMAT(*  FAILED TO CONVERGE ON T(*, I2, *), T = *,
     1 C(E14.6,E14.6),* LAST CORRECTION =*, C(E14.6,E14.6))
      RETURN
      END
```

```
      SUBROUTINE GUDFAZ(K,C,FAZ,N)
C
C             SMOOTHS PHASE OF IONO REFLECTION COEFFICIENTS FOR ANIHOP
C             K=1,2,3,4 FOR TEE,TEM,TME,TMM, RESPECTIVELY
C             C=COS(PHI),  ARRANGED IN ORDER OF INCREASING PHI
C                           (=ANGLE OF INCIDENCE)
C             FAZ=PHASE OF REFLECTION COEFFICIENT, ARRANGED IN
C                           SAME ORDER AS C.
C             N=NUMBER OF DATA POINTS
C
C
      DIMENSION C(20),FAZ(4,20)
      DATA (PI=3.141592653)
      C(N+1)=0.
      FAZ(K,N+1)=PI
    3 IF(ABS(PI-FAZ(K,N)) .LE. PI)  GO TO 5
      FAZ(K,N)=FAZ(K,N)-SIGNF(2.*PI,FAZ(K,N))
      GO TO 3
    5 DO 10  I=2,N
      J=N+2-I
      TEST=PI+(FAZ(K,J+1)-FAZ(K,J))*C(J-1)/(C(J+1)-C(J))
    7 TRY=TEST-FAZ(K,J-1)
      IF(ABS(TRY) .LE. PI) GO TO 8
      FAZ(K,J-1)=FAZ(K,J-1)+SIGNF(2.*PI,TRY)
      GO TO 7
    8 TRY = FAZ(K,J-1) -FAZ(K,J)
      IF(ABS(TRY) .LT. 2.*PI) GO TO 10
      FAZ(K,J-1) = FAZ(K,J-1) - SIGNF( 2.*PI, TRY)
      GO TO 8
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CWAIRY(KK,T,F1,M1,F2,M2)
C     CALCULATION OF THE W(T) AIRY FUNCTIONS
C     INPUT
C        KK=1, W(T) OF KIND 1 AND W(T) OF KIND 2 ARE COMPUTED
C        KK=2, THE DERIVATIVE OF W(T) OF KIND 1 AND THE DERIVATIVE OF
C        W(T) OF KIND 2 ARE COMPUTED
C        T = THE COMPLEX ARGUMENT
C     OUTPUT
C        F1*(E**M1) = W(T) OF KIND 1 OR THE DERIVATIVE OF W(T) OF KIND 1
C        AS INDICATED BY KK
C        F2*(E**M2) = W(T) OF KIND 2 OR THE DERIVATIVE OF W(T) OF KIND 2
C        AS INDICATED BY KK
C     NOTE.  F1 AND F2 ARE COMPLEX, E=2.718281828..., AND M1 AND M2 ARE
C     EXPONENTS

      COMMON/MEXP/M
      COMPLEX F1,F2,WI1,WI2,WI1P,WI2P,T
      GO TO(100,200),KK
  100 F2=WI1(T)
      M2=M
      F1=WI2(T)
      M1=M
      GO TO 300
  200 F2=WI1P(T)
      M2=M
      F1=WI2P(T)
      M1=M
  300 F1=1.7724538509*(0.,-1.)*F1
      F2=1.7724538509*(0.,+1.)*F2
      RETURN
      END
```

```
      COMPLEX FUNCTION AIRY(ZZ)
C
C          SUBROUTINE TO CALCULATE AIRY FUNCTIONS.  USES TAYLORS EXPANSIONS
C          AROUND VARIOUS COMPLEX CONSTANTS FOR SMALL T AND ASSYMPTOTIC
C          SERIES  FOR LARGE T.  WRITTEN BY DR. GEORGE HUFFORD,  ITS,
C          BOULDER, COLORADO  80302.
C
      COMPLEX ZZ
      COMMON/MEXP/M
C
      COMPLEX Z,Z1,A,AP,U,ZT,ZA,ZB,ZE,ZR,B0,B1,B2,B3
      DIMENSION X(2),X1(2),XT(2) $ EQUIVALENCE (X,Z),(X1,Z1),(XT,ZT)
      DATA (LG=3),(Z1=(0.,0.)),(A=(0.35502805389,0.))
      DATA (AP=(-0.25881940379,0.))
      DIMENSION   AV( 70) $ COMPLEX   AV $ DATA (   AV=                 AV   0
     X  (-3.2914517363E-001, 0.0000000000E+000),                        AV   1
     X  (-2.6780035625E+000, 1.4774589547E+000),                        AV   2
     X  ( 3.5076100903E-001, 0.0000000000E+000),                        AV   3
     X  ( 2.4122262158E+000, 6.9865124448E-001),                        AV   4
     X  ( 3.3635531189E+001,-3.4600959696E+000),                        AV   5
     X  ( 3.4449739613E+002,-3.3690890250E+002),                        AV   6
     X  (-7.0265532950E-002, 0.0000000000E+000),                        AV   7
     X  (-5.4818219290E-001,-1.9207365909E+000),                        AV   8
     X  (-1.3383395342E+001,-1.6022590802E+001),                        AV   9
     X  (-2.2967795901E+002,-3.2072452637E+001),                        AV  10
     X  (-1.8040780476E+003, 2.1917675036E+003),                        AV  11
     X  (-3.7881429368E-001, 0.0000000000E+000),                        AV  12
     X  (-1.3491836060E+000, 8.4969077213E-001),                        AV  13
     X  (-6.0453339320E+000, 1.0623175540E+001),                        AV  14
     X  ( 3.1169621695E+001, 9.8813517650E+001),                        AV  15
     X  ( 9.8925349347E+002, 1.3905286008E+002),                        AV  16
     X  ( 2.2740742820E-001, 0.0000000000E+000),                        AV  17
     X  ( 7.1857403459E-001, 9.7809094170E-001),                        AV  18
     X  ( 6.0621088063E+000, 2.7203014866E+000),                        AV  19
     X  ( 3.6307084828E+001,-2.0961355813E+001),                        AV  20
     X  (-6.7139789190E+001,-3.0904638708E+002),                        AV  21
     X  (-2.8001653691E+003, 4.6649365984E+002),                        AV  22
     X  ( 5.3556088329E-001, 0.0000000000E+000),                        AV  23
     X  ( 9.2407365385E-001,-1.9106560052E-001),                        AV  24
     X  ( 1.8716185961E+000,-2.5743310394E+000),                        AV  25
     X  (-7.2188436328E+000,-1.2924200190E+001),                        AV  26
     X  (-8.1787377840E+001, 3.2087013839E+001),                        AV  27
     X  ( 2.9933948552E+002, 5.6922179258E+002),                        AV  28
     X  ( 3.5502805389E-001, 0.0000000000E+000),                        AV  29
     X  ( 3.1203438104E-001,-3.8845385098E-001),                        AV  30
     X  (-5.2839999360E-001,-1.0976411220E+000),                        AV  31
     X  (-4.2009351585E+000, 1.1940151191E+000),                        AV  32
     X  ( 7.1858832892E+000, 1.9600912513E+001),                        AV  33
     X  ( 1.0129121011E+002,-7.5951233292E+001),                        AV  34
     X  ( 1.3529241631E-001, 0.0000000000E+000),                        AV  35
     X  ( 3.2618478398E-002,-1.7084872785E-001),                        AV  36
     X  (-3.4215381085E-001,-8.9067646330E-002),                        AV  37
     X  (-1.4509641493E-001, 1.0328015748E+000),                        AV  38
     X  ( 4.1001968523E+000,-6.8936911760E-001),                        AV  39
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```
X  (-1.3030124036E+001,-1.6910541453E+001),                       AV  40
X  ( 3.4924130423E-002, 0.0000000000E+000),                       AV  41
X  (-8.4464726625E-003,-4.2045154421E-002),                       AV  42
X  (-6.9313268963E-002, 3.5364798705E-002),                       AV  43
X  ( 1.5227622646E-001, 1.2848454470E-001),                       AV  44
X  ( 1.0681373184E-001,-6.7766153503E-001),                       AV  45
X  (-2.6193432727E+000, 1.5699859905E+000),                       AV  46
X  ( 6.5911393574E-003, 0.0000000000E+000),                       AV  47
X  (-3.9443985580E-003,-6.8060106117E-003),                       AV  48
X  (-5.9820131079E-003, 1.1799010149E-002),                       AV  49
X  ( 2.9922498406E-002,-5.9772930737E-003),                       AV  50
X  (-7.7464130231E-002,-5.2292402759E-002),                       AV  51
X  ( 1.1276585896E-001, 3.5112442431E-001),                       AV  52
X  ( 9.5156385121E-004, 0.0000000000E+000),                       AV  53
X  (-8.0842995655E-004,-7.6590132690E-004),                       AV  54
X  ( 1.6147816065E-004, 1.7661755136E-003),                       AV  55
X  ( 2.0138718363E-003,-3.1976716632E-003),                       AV  56
X  (-9.5086784440E-003, 4.5377832492E-003),                       AV  57
X  ( 3.7560191819E-002, 5.7361916864E-004),                       AV  58
X  ( 1.0834442814E-004, 0.0000000000E+000),                       AV  59
X  (-1.0968606480E-004,-5.9902329668E-005),                       AV  60
X  ( 1.0778191327E-004, 1.5771596227E-004),                       AV  61
X  (-6.8980937889E-005,-3.7626457370E-004),                       AV  62
X  (-1.6166126174E-004, 9.7457773280E-004),                       AV  63
X  ( 9.9476943603E-006, 0.0000000000E+000),                       AV  64
X  (-1.0956823939E-005,-2.9508799638E-006),                       AV  65
X  ( 1.4709074502E-005, 8.1042089702E-006),                       AV  66
X  (-2.4446015180E-005,-2.0638143108E-005),                       AV  67
X  ( 7.4921288640E-007, 0.0000000000E+000),                       AV  68
X  (-8.4619068946E-007,-3.6807338399E-008),                       AV  69
X  ( 1.2183963384E-006, 8.3589199402E-008))                       AV  70
 DIMENSION  APV( 70) $ COMPLEX  APV $ DATA ( APV=                APV   0
X  ( 3.4593548728E-001, 0.0000000000E+000),                      APV   1
X  ( 4.1708876594E+000, 6.2414437707E+000),                      APV   2
X  ( 3.2719281855E-001, 0.0000000000E+000),                      APV   3
X  ( 1.0828742735E+000,-5.4928302529E+000),                      APV   4
X  (-2.3363517933E+001,-7.4901848140E+001),                      APV   5
X  (-1.0264877579E+003,-5.6707940802E+002),                      APV   6
X  (-7.9062857537E-001, 0.0000000000E+000),                      APV   7
X  (-3.8085833358E+000, 1.5129605192E+000),                      APV   8
X  (-2.6086379081E+001, 3.5540709915E+001),                      APV   9
X  ( 1.0761838222E+002, 5.1239944904E+002),                      APV  10
X  ( 6.6597797197E+003, 1.8096186253E+003),                      APV  11
X  ( 3.1458376921E-001, 0.0000000000E+000),                      APV  12
X  ( 1.8715425470E+000, 2.0544836557E+000),                      APV  13
X  ( 2.2591736932E+001, 4.8562995474E+000),                      APV  14
X  ( 1.6162997879E+002,-1.4335597185E+002),                      APV  15
X  (-8.0047161665E+002,-2.1527454270E+003),                      APV  16
X  ( 6.1825902074E-001, 0.0000000000E+000),                      APV  17
X  ( 1.3019603890E+000,-1.2290774954E+000),                      APV  18
X  ( 1.5036118745E-001,-1.1008092874E+001),                      APV  19
X  (-7.0116800393E+001,-4.0480822759E+001),                      APV  20
X  (-4.8317166902E+002, 4.9692755718E+002),                      APV  21
X  ( 4.8970655652E+003, 4.8627290801E+003),                      APV  22
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X  (-1.0160567116E-002, 0.0000000000E+000),                      APV  23
X  (-5.4826636454E-001,-7.1365288463E-001),                      APV  24
X  (-4.6749134088E+000,-1.1924245293E-001),                      APV  25
X  (-1.0536397828E+001, 2.4943711387E+001),                      APV  26
X  ( 1.6333770696E+002, 9.0394910688E+001),                      APV  27
X  ( 5.6449455285E+002,-1.4248324426E+003),                      APV  28
X  (-2.5881940380E-001, 0.0000000000E+000),                      APV  29
X  (-4.8620754109E-001, 1.5689924913E-001),                      APV  30
X  (-4.7348131897E-001, 1.7093438130E+000),                      APV  31
X  ( 7.0373840765E+000, 3.6281824918E+000),                      APV  32
X  ( 1.7739586372E+001,-4.0360422402E+001),                      APV  33
X  (-2.9791511956E+002,-3.8408892977E+001),                      APV  34
X  (-1.5914744130E-001, 0.0000000000E+000),                      APV  35
X  (-1.1340423572E-001, 1.9730504925E-001),                      APV  36
X  ( 4.0126209154E-001, 3.9222995820E-001),                      APV  37
X  ( 1.3348652430E+000,-1.4377272421E+000),                      APV  38
X  (-7.9022494720E+000,-4.2063644605E+000),                      APV  39
X  (-1.3892752107E+000, 5.1229416787E+001),                      APV  40
X  (-5.3090384434E-002, 0.0000000000E+000),                      APV  41
X  (-1.6832965528E-003, 6.8366967859E-002),                      APV  42
X  ( 1.3789401334E-001,-1.1613804016E-002),                      APV  43
X  (-1.4713730621E-001,-3.7151985747E-001),                      APV  44
X  (-1.0070196495E+000, 1.1591348425E+000),                      APV  45
X  ( 7.5045049133E+000, 4.6913115383E-001),                      APV  46
X  (-1.1912976706E-002, 0.0000000000E+000),                      APV  47
X  ( 5.1468574932E-003, 1.3660891236E-002),                      APV  48
X  ( 1.8309710537E-002,-1.8808588497E-002),                      APV  49
X  (-6.4461593156E-002,-1.3611794718E-002),                      APV  50
X  ( 1.0516239905E-001, 1.9313053560E-001),                      APV  51
X  ( 2.0520046212E-001,-9.1772617372E-001),                      APV  52
X  (-1.9586409502E-003, 0.0000000000E+000),                      APV  53
X  ( 1.4695649526E-003, 1.8086384633E-003),                      APV  54
X  ( 5.9709947951E-004,-3.8332699216E-003),                      APV  55
X  (-6.8910893004E-003, 5.4467425272E-003),                      APV  56
X  ( 2.6167927738E-002,-8.4092000294E-004),                      APV  57
X  (-8.8284474192E-002,-4.6475312179E-002),                      APV  58
X  (-2.4741389087E-004, 0.0000000000E+000),                      APV  59
X  ( 2.3707837404E-004, 1.6461109527E-004),                      APV  60
X  (-1.7465569860E-004,-4.2026783985E-004),                      APV  61
X  (-1.0394516180E-004, 9.4761844375E-004),                      APV  62
X  ( 1.3004110522E-003,-2.2446656813E-003),                      APV  63
X  (-2.4765200397E-005, 0.0000000000E+000),                      APV  64
X  ( 2.6714870982E-005, 9.8691565027E-006),                      APV  65
X  (-3.3539774178E-005,-2.7113284942E-005),                      APV  66
X  ( 4.9197843140E-005, 6.9349092091E-005),                      APV  67
X  (-2.0081508947E-006, 0.0000000000E+000),                      APV  68
X  ( 2.2671244519E-006, 2.7848508382E-007),                      APV  69
X  (-3.2692132725E-006,-7.3943488682E-007))                      APV  70
 DIMENSION ASLT( 17) $ DATA (ASLT= 1.1407E+002, 1.1549E+002,    ASLT   0
X    1.1779E+002, 1.2124E+002, 1.2619E+002, 1.3319E+002,        ASLT   1
X    1.4307E+002, 1.5716E+002, 1.7774E+002, 2.0884E+002,        ASLT   2
X    2.5832E+002, 3.4294E+002, 5.0339E+002, 8.5678E+002,        ASLT   3
X    1.8336E+003, 5.7270E+003, 3.5401E+004)                      ASLT   4
 DIMENSION  ASV( 21) $ DATA ( ASV= 1.8335766942E+010,            ASV   0
```

```
   X   1.9293755496E+009, 2.1428803701E+008, 2.5198919876E+007,          ASV   1
   X   3.1482574185E+006, 4.1952487519E+005, 5.9892513580E+004,          ASV   2
   X   9.2072066015E+003, 1.5331694323E+003, 2.7846508084E+002,          ASV   3
   X   5.5622785377E+001, 1.2341573335E+001, 3.0794530307E+000,          ASV   4
   X   8.7766696967E-001, 2.9159139927E-001, 1.1609906404E-001,          ASV   5
   X   5.7649190421E-002, 3.7993059132E-002, 3.7133487657E-002,          ASV   6
   X   6.9444444448E-002, 1.0000000000E+000)                             ASV   7
    DIMENSION APSV( 21) $ DATA (APSV=-1.8643931093E+010,                APSV   0
   X  -1.9635237894E+009,-2.1829342088E+008,-2.5697908389E+007,         APSV   1
   X  -3.2145365220E+006,-4.2895240048E+005,-6.1335706678E+004,         APSV   2
   X  -9.4463548250E+003,-1.5763573037E+003,-2.8703323717E+002,         APSV   3
   X  -5.7508303524E+001,-1.2807293083E+001,-3.2104935853E+000,         APSV   4
   X  -9.2047999257E-001,-3.0825376496E-001,-1.2410589605E-001,         APSV   5
   X  -6.2662163500E-002,-4.2462830794E-002,-4.3885030868E-002,         APSV   6
   X  -9.7222222227E-002, 1.0000000000E+000)                            APSV   7
    DIMENSION NQTT(15),NQT(8) $ EQUIVALENCE (NQTT(8),NQT(1))
    DATA(NQTT=1,3,7,12,17,23,29,35,41,47,53,59,64,68,71)
    ANM(Z)=ABSF(REAL(Z))+ABSF(AIMAG(Z))
    ENTRY AI $ LA=0 $ GO TO 1
    ENTRY AIP $ LA=0 $ GO TO 2
    ENTRY WI $ ENTRY WI1 $ LA=1 $ GO TO 1
    ENTRY WIP $ ENTRY WI1P $ LA=1 $ GO TO 2
    ENTRY WI2 $ LA=-1 $ GO TO 1
    ENTRY WI2P $ LA=-1 $ GO TO 2
  1 LB=0 $ GO TO 3
  2 LB=1 $ GO TO 3
  3 Z=ZZ
    IF(LA) 5,7,4
  4 U=(-0.5,0.86602540378) $ GO TO 6
  5 U=(-0.5,-0.86602540378)
  6 Z=U*Z
  7 LC=0 $ IF(X(2)) 8,9,10
  8 LC=1 $ X(2)=-X(2) $ GO TO 10
  9 X(2)=0.
 10 CONTINUE     COMPARE WITH PREVIOUS
    IF(X(1) .NE. X1(1) .OR. X(2) .NE. X1(2)) GO TO 20
    I=LB+1 $ I=LG.AND.I $ IF(I) GO TO 400
    IF(LB) 220,210
400 CONTINUE     EXIT
    IF(LB) 402,401
401 ZT=A $ IF(LC)XT(2)=-XT(2)
    IF(LA)404,411,403
402 ZT=AP $ IF(LC)XT(2)=-XT(2)
    IF(LA)403,411,404
403 U=(1.,-1.7320508076) $ GO TO 410
404 U=(1.,1.7320508076) $ GO TO 410
410 ZT=U*ZT
411 AIRY=ZT $ RETURN
 20 CONTINUE                AFFINE COORDINATES
    M=0
    Z1=Z $ LG=0
    IF(X(1).LE.-7. .OR. X(1).GT.7. .OR. X(2).GT.6.928203232) GO TO 200
    IP=7.-X(1) $ P=IP=7-IP
    Q=IQ=0.86602540378*X(2)+0.5*(P-X(1))
```

```
      N=NQT(IP)+IQ
      IF(N.GE.NQT(IP+1)) GO TO 200
  100 CONTINUE         SERIES
      XT(1)=P $ XT(2)=1.1547005384*Q
      U=Z-ZT $ B1=AV(N) $ B3=B1*ZT*U $ AP=APV(N) $ B2=AP*U
      A=B2+B1 $ AP=AP+B3
      AN=1.
      DO 110 I=2,3
      AN=AN+1.
      B3=B3*U/AN
      A=B3+A
      B0=B1 $ B1=B2 $ B2=B3
      B3=(ZT*B1+U*B0)*U/AN
      AP=B3+AP
      IF(ANM(B2).GT.0.5E-10*ANM(A) .AND. ANM(B3).GT.0.5E-10*ANM(AP)) I=0
  110 CONTINUE
      LG=3
      GO TO 400
  200 CONTINUE            ASYMPTOTICS
      ZA=CSQRT(Z)
      ZB=0.28209479177/CSQRT(ZA)
      ZT=-0.66666666667*Z*ZA
      T=XT(1)**2+XT(2)**2
      CALL ZEXP(XT(1),XT(2),SX,SY,M)
      ZE=CMPLX(SX,SY)
      ZM=2.718281828**(M+M)
      ZR=1./ZT
      IF(XT(2) .GT. 0. .AND. XT(1) .LT. 11.8595) LG=4
      DO 201 NT=2,18
      IF(T .LT. ASLT(NT-1)) GO TO 202
  201 CONTINUE $ NT=19
  202 IF(LB) 220,210
  210 CONTINUE              A
      ZT=ASV(NT-1)
      DO 211 I=NT,21
  211 ZT=ASV(I)+ZT*ZR
      A=ZT*ZE
      I=4.AND.LG $ IF(I) 212,216
  212 ZT=ASV(NT-1)
      DO 213 I=NT,21
  213 ZT=ASV(I)-ZT*ZR
      A=A+(0.,1.)*ZT/(ZE*ZM)
  216 A=ZB*A $ LG=1.OR.LG $ GO TO 401
  220 CONTINUE           AP
      ZT=APSV(NT-1)
      DO 221 I=NT,21
  221 ZT=APSV(I)+ZT*ZR
      AP=-ZT*ZE
      I=4.AND.LG $ IF(I) 222,226
  222 ZT=APSV(NT-1)
      DO 223 I=NT,21
  223 ZT=APSV(I)-ZT*ZR
      AP=AP+(0.,1.)*ZT/(ZE*ZM)
  226 AP=ZA*ZB*AP $ LG=2.OR.LG $ GO TO 402
      END
```

7. APPENDIX: AN IONOSPHERIC REFLECTION COEFFICIENT PROGRAM

7.1 General Description

The propagation program described in the main body of this report requires ionospheric reflection coefficients as input. We will now describe briefly a computer program that can be used to calculate such reflection coefficients for any profile of ionospheric electron density.

We assume a plane ionosphere whose electron density and electron-neutral collision frequency vary only with altitude and a static magnetic field with constant dip angle and magnitude. For an infinite plane wave incident from below on the ionosphere at an angle, φ, we compute the reflection coefficient matrix defined by

$$\begin{bmatrix} {}_rE_e \\ {}_rE_m \end{bmatrix} = \begin{bmatrix} T_{ee} & T_{me} \\ T_{em} & T_{mm} \end{bmatrix} \cdot \begin{bmatrix} {}_iE_e \\ {}_iE_m \end{bmatrix} \qquad \text{(A1)}$$

where the pre-subscript indicates the incident or reflected wave and the post-subscript indicates whether the electric or magnetic vector is in the plane of incidence (e is "vertical polarization," m is "horizontal polarization").

The solution is accomplished by (1) dividing the ionosphere into thin homogeneous slabs whose properties are those of the continuous profile at the center of the slab, (2) solving Maxwell's equations in each slab, and (3) satisfying the boundary conditions at the slab interfaces. The solution is derived by Johler and Harper (1962).

The user must determine the height profile of electron density, the magnetic field parameters (strength, dip angle, azimuth relative to direction of propagation), the radio frequency, and the angle of incidence. By modifying the appropriate subroutine, he may specify the collision frequency.

The magnetic field parameters are well known (certainly to the precision required for this application), and maps of them can be found for instance in Davies (1965).

The collision frequency is also fairly well known--it decreases almost exponentially with height with a slope that the experts agree on. There is about a factor of 3 disagreement on the absolute magnitude at a given height (or an uncertainty of about 3 km in the height for a given value). The collision frequency computed in the subroutine CFEO is that recommended by Gambill and Knapp (1969). You can change subroutine CFEO to return any collision frequency profile you want.

We do not know the height distribution of electrons in the lower ionosphere very well yet; nor do we know how the electron density varies with season, sunspot cycle, latitude, etc. So we have included two versions of subroutine ENN to define electron density as a function of height. One simply interpolates in a table supplied as input by the user. The other contains a crude model ionosphere, which varies with the sun's zenith angle. This latter model (described by Berry and Jones (1970)) does a fair job of predicting the LF and VLF field strength variations during the day, especially at middle latitudes. You can easily incorporate any model of the ionosphere you want into the calculation by writing an appropriate version of ENN and substituting it for the version listed in this report.

Figure 14 shows a large block diagram of the organization of the program ANIREF. Details of the implementation can be determined from the comments cards in the FORTRAN listing at the end of this appendix.

7.2 A Sample Case

Table 2 lists the input parameters required by the program. The input for a sample case is shown in figure 15. The block diagram in

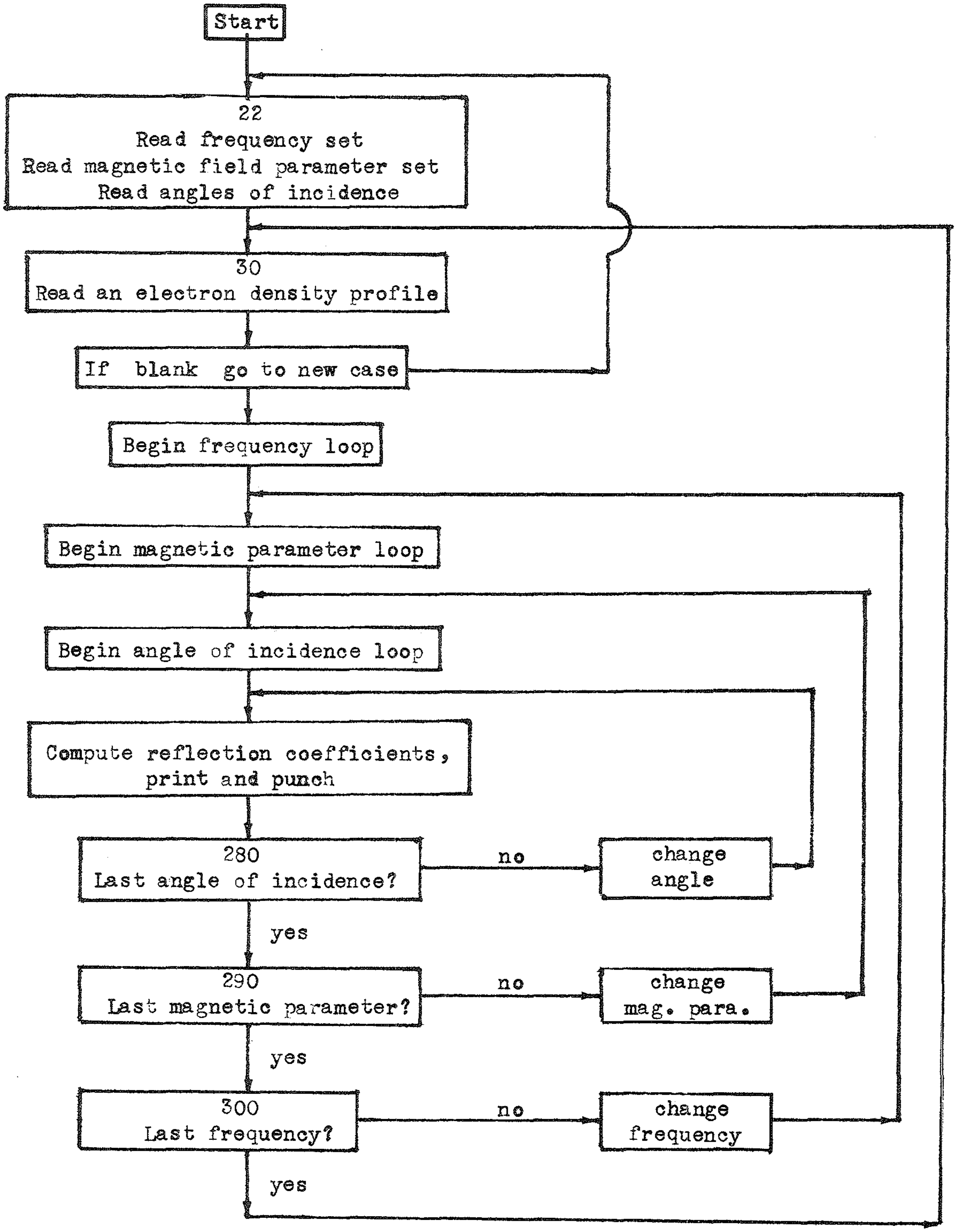
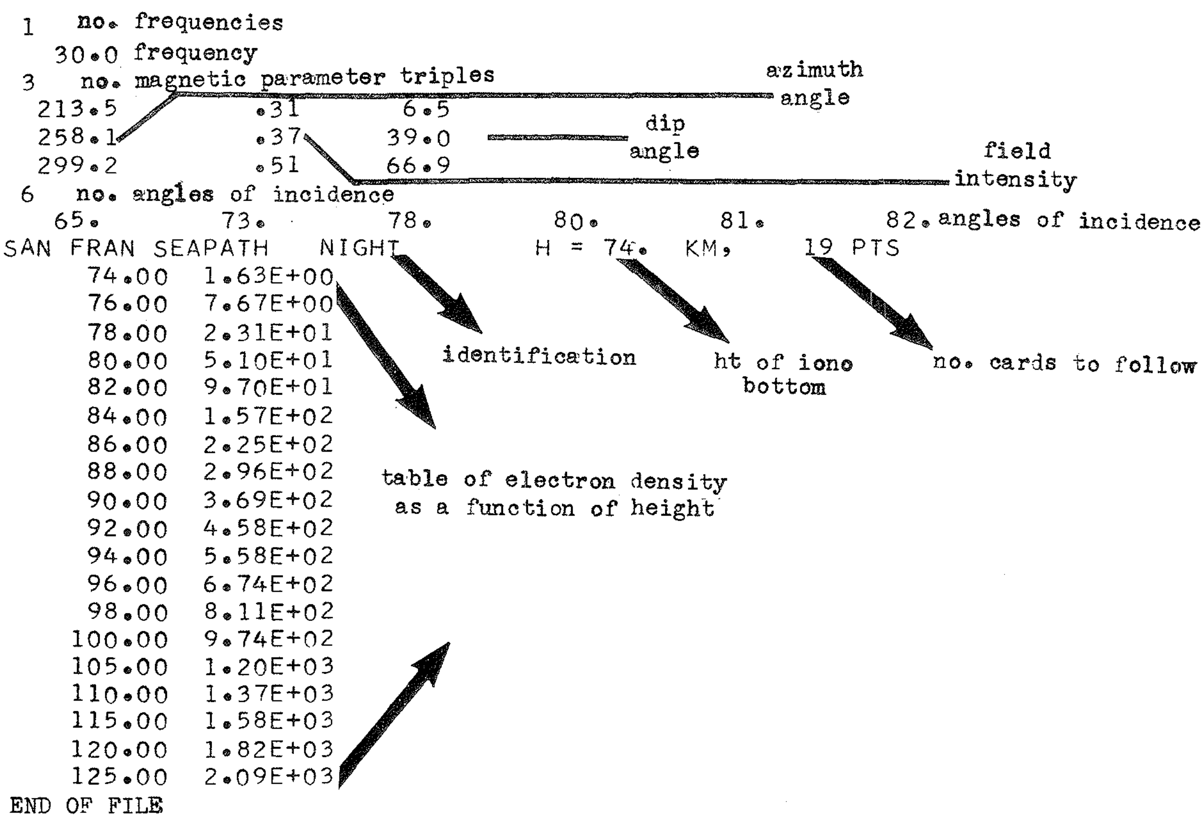


Figure 14. Flow chart for Program ANIREF.

Table 2. Input Data for Program ANIREF

Variable	Physical Quantity	Units	Comments
IFR	Number		Number of frequencies to be read in; maximum of 9.
FRE(I)	Radio frequency	kHz	
NPA	Number		Number of sets of magnetic field parameters; maximum of 8.
PHA(I)	Angle	Degrees	Azimuth of propagation of incident wave measured clockwise from north.
HA(I)	Magnetic strength	Gauss	Earth's magnetic field intensity.
DAP(I)	Dip	Degrees	Magnetic field dip angle, measured from horizontal.
NPI	Number		Number of angles of incidence; maximum of 10.
SPHI(I)	Angle	Degrees	Angle of incidence of radio wave on ionosphere; measured from vertical.
PROFLE(I)			Alphanumeric identification of ionospheric model.
HITE	Height	km	Height of bottom of ionosphere. Should equal Z(1) if ionosphere is defined by a table.
LAYR	Number		Number of data cards defining ionospheric model.
Z(L)	Height	km	Usually a table defining electron density as a function of height. Can also be used to define parameters of an analytic model of electron density.
EN(L)	Electron density	electrons per cc	

ANIREF INPUT



```
1   no. frequencies
  30.0 frequency
3   no. magnetic parameter triples
 213.5          .31      6.5
 258.1          .37     39.0
 299.2          .51     66.9
6   no. angles of incidence
  65.        73.        78.        80.        81.        82. angles of incidence
SAN FRAN SEAPATH   NIGHT         H = 74.  KM,    19 PTS
     74.00   1.63E+00
     76.00   7.67E+00
     78.00   2.31E+01
     80.00   5.10E+01
     82.00   9.70E+01
     84.00   1.57E+02
     86.00   2.25E+02
     88.00   2.96E+02
     90.00   3.69E+02
     92.00   4.58E+02
     94.00   5.58E+02
     96.00   6.74E+02
     98.00   8.11E+02
    100.00   9.74E+02
    105.00   1.20E+03
    110.00   1.37E+03
    115.00   1.58E+03
    120.00   1.82E+03
    125.00   2.09E+03
END OF FILE
```

Figure 15. List of input data deck for ANIREF sample case.

figure 16 shows possibilities of data deck set up when several cases are to be run at one time.

The printed output for this sample case is listed in figures 17 and 18. The punched cards produced by this sample case are listed in figure 19. These cards are in the right format to be used as input to ANIHOP and were used as input for the sample case in the main body of this report (compare fig. 4).

The input electron density profile is printed on the first page of output (fig. 17). Then a table of computed reflection coefficients is printed under a heading which contains the other relevant input information. Comments have been superimposed on the printout to help explain the various entries.

7.3 Mathematical Details of Reflection Coefficient Calculation

The main subroutine of ANIREF solves the set of equations

$$
\left[\begin{array}{l}
a_{1,1}\ a_{1,2}\ a_{1,3}\ a_{1,4}\ a_{1,5}\ a_{1,6} \\
b_{1,1}\ b_{1,2}\ b_{1,3}\ b_{1,4}\ b_{1,5}\ b_{1,6} \\
c_{1,1}\ c_{1,2}\ c_{1,3}\ c_{1,4}\ c_{1,5}\ c_{1,6} \\
d_{1,1}\ d_{1,2}\ d_{1,3}\ d_{1,4}\ d_{1,5}\ d_{1,6} \\
\qquad\qquad a_{2,3}\ a_{2,4}\ a_{2,5}\ a_{2,6}\ a_{2,7}\ a_{2,8}\ a_{2,9}\ a_{2,10} \\
\qquad\qquad b_{2,3}\ b_{2,4}\ b_{2,5}\ b_{2,6}\ b_{2,7}\ b_{2,8}\ b_{2,9}\ b_{2,10} \\
\qquad\qquad c_{2,3}\ c_{2,4}\ c_{2,5}\ c_{2,6}\ c_{2,7}\ c_{2,8}\ c_{2,9}\ c_{2,10} \\
\qquad\qquad d_{2,3}\ d_{2,4}\ d_{2,5}\ d_{2,6}\ d_{2,7}\ d_{2,8}\ d_{2,9}\ d_{2,10} \\
\qquad\qquad\qquad\qquad\qquad\quad a_{3,7}\ a_{3,8}\ a_{3,9}\ a_{3,10}\ a_{3,11}\ a_{3,12}\ a_{3,13}\ a_{3,14} \\
\qquad\qquad\qquad\qquad\qquad\quad b_{3,7}\ b_{3,8}\ b_{3,9}\ b_{3,10}\ b_{3,11}\ b_{3,12}\ b_{3,13}\ b_{3,14} \\
\qquad\qquad\qquad\qquad\qquad\quad c_{3,7}\ c_{3,8}\ c_{3,9}\ c_{3,10}\ c_{3,11}\ c_{3,12}\ c_{3,13}\ c_{3,14} \\
\qquad\qquad\qquad\qquad\qquad\quad d_{3,7}\ d_{3,8}\ d_{3,9}\ d_{3,10}\ d_{3,11}\ d_{3,12}\ d_{3,13}\ d_{3,14} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad a_{p,4p-5}\ \cdots\cdots\cdots\cdots\ a_{p,4p} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad b_{p,4p-5}\ \cdots\cdots\cdots\cdots\ b_{p,4p} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad c_{p,4p-5}\ \cdots\cdots\cdots\cdots\ c_{p,4p} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad d_{p,4p-5}\ \cdots\cdots\cdots\cdots\ d_{p,4p}
\end{array}\right]
\left[\begin{array}{l}
T_{em},\ T_{mm} \\
T_{ee},\ T_{me} \\
U^{(1)}_{eio},\ U^{(1)}_{mio} \\
U^{(1)}_{eie},\ U^{(1)}_{mie} \\
U^{(1)}_{ero},\ U^{(1)}_{mro} \\
U^{(1)}_{ere},\ U^{(1)}_{mre} \\
U^{(2)}_{eio},\ U^{(2)}_{mio} \\
U^{(2)}_{eie},\ U^{(2)}_{mie} \\
U^{(2)}_{ero},\ U^{(2)}_{mro} \\
U^{(2)}_{ere},\ U^{(2)}_{mre} \\
\quad\cdot\qquad\cdot \\
\quad\cdot\qquad\cdot \\
\quad\cdot\qquad\cdot \\
\quad\cdot\qquad\cdot \\
U^{(p-1)}_{eio},\ U^{(p-1)}_{mio} \\
U^{(p-1)}_{eie},\ U^{(p-1)}_{mie} \\
U^{(p-1)}_{ero},\ U^{(p-1)}_{mro} \\
U^{(p-1)}_{ere},\ U^{(p-1)}_{mre} \\
U^{(p)}_{eio},\ U^{(p)}_{mio} \\
U^{(p)}_{eie},\ U^{(p)}_{mie}
\end{array}\right]
+
\left[\begin{array}{l}
a_{oe},\ a_{om} \\
b_{oe},\ b_{om} \\
c_{oe},\ c_{om} \\
d_{oe},\ d_{om} \\
\\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\
\end{array}\right]
=0, \tag{A2}
$$

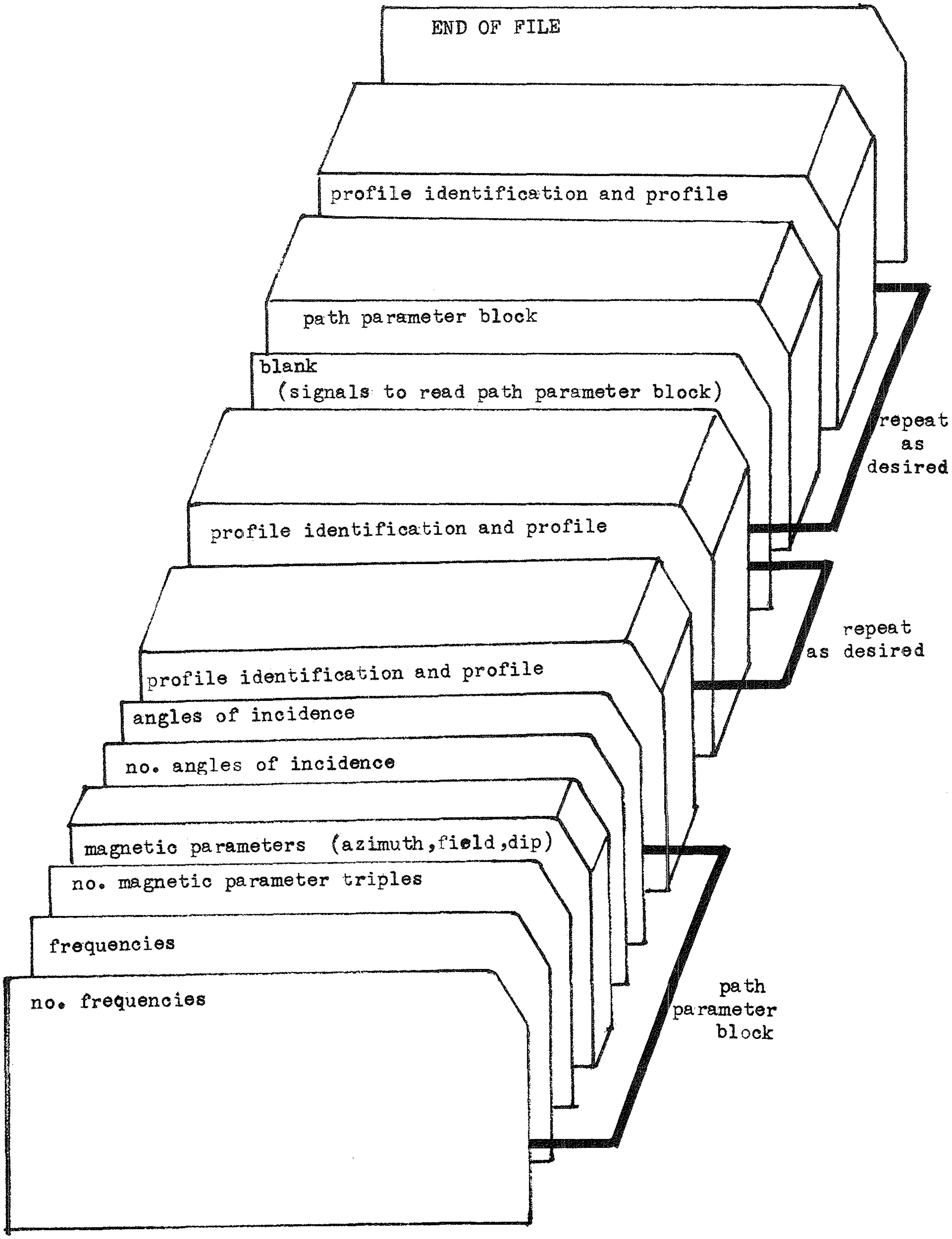


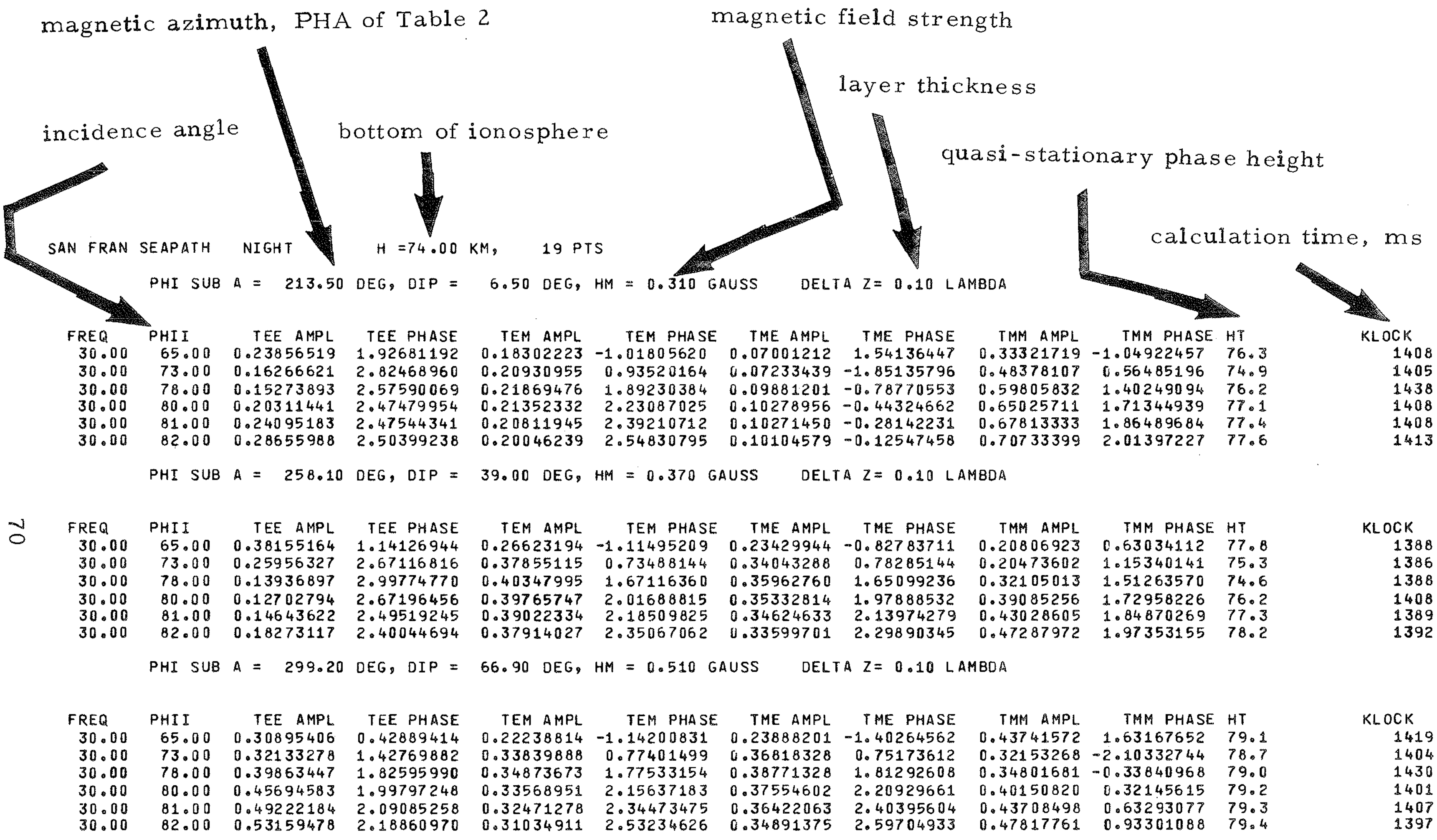
Figure 16. Data deck setup for Program ANIREF.

```
                    print out of input profile

SAN FRAN SEAPATH   NIGHT        H =74.00 KM,    19 PTS
     74.00  1.63+000
     76.00  7.67+000
     78.00  2.31+001
     80.00  5.10+001
     82.00  9.70+001
     84.00  1.57+002
     86.00  2.25+002
     88.00  2.96+002
     90.00  3.69+002
     92.00  4.58+002
     94.00  5.58+002
     96.00  6.74+002
     98.00  8.11+002
    100.00  9.74+002
    105.00  1.20+003
    110.00  1.37+003
    115.00  1.58+003
    120.00  1.82+003
    125.00  2.09+003
```

h N(h), per cc

Figure 17. Printed output from ANIREF sample case, page 1.



magnetic azimuth, PHA of Table 2

magnetic field strength

layer thickness

incidence angle

bottom of ionosphere

quasi-stationary phase height

calculation time, ms

```
SAN FRAN SEAPATH    NIGHT          H =74.00 KM,    19 PTS

          PHI SUB A =  213.50 DEG, DIP =   6.50 DEG, HM = 0.310 GAUSS    DELTA Z= 0.10 LAMBDA

 FREQ     PHII       TEE AMPL   TEE PHASE    TEM AMPL    TEM PHASE   TME AMPL    TME PHASE    TMM AMPL    TMM PHASE HT          KLOCK
  30.00    65.00  0.23856519  1.92681192  0.18302223 -1.01805620  0.07001212  1.54136447  0.33321719 -1.04922457  76.3          1408
  30.00    73.00  0.16266621  2.82468960  0.20930955  0.93520164  0.07233439 -1.85135796  0.48378107  0.56485196  74.9          1405
  30.00    78.00  0.15273893  2.57590069  0.21869476  1.89230384  0.09881201 -0.78770553  0.59805832  1.40249094  76.2          1438
  30.00    80.00  0.20311441  2.47479954  0.21352332  2.23087025  0.10278956 -0.44324662  0.65025711  1.71344939  77.1          1408
  30.00    81.00  0.24095183  2.47544341  0.20811945  2.39210712  0.10271450 -0.28142231  0.67813333  1.86489684  77.4          1408
  30.00    82.00  0.28655988  2.50399238  0.20046239  2.54830795  0.10104579 -0.12547458  0.70733399  2.01397227  77.6          1413

          PHI SUB A =  258.10 DEG, DIP =  39.00 DEG, HM = 0.370 GAUSS    DELTA Z= 0.10 LAMBDA

 FREQ     PHII       TEE AMPL   TEE PHASE    TEM AMPL    TEM PHASE   TME AMPL    TME PHASE    TMM AMPL    TMM PHASE HT          KLOCK
  30.00    65.00  0.38155164  1.14126944  0.26623194 -1.11495209  0.23429944 -0.82783711  0.20806923  0.63034112  77.8          1388
  30.00    73.00  0.25956327  2.67116816  0.37855115  0.73488144  0.34043288  0.78285144  0.20473602  1.15340141  75.3          1386
  30.00    78.00  0.13936897  2.99774770  0.40347995  1.67116360  0.35962760  1.65099236  0.32105013  1.51263570  74.6          1388
  30.00    80.00  0.12702794  2.67196456  0.39765747  2.01688815  0.35332814  1.97888532  0.39085256  1.72958226  76.2          1408
  30.00    81.00  0.14643622  2.49519245  0.39022334  2.18509825  0.34624633  2.13974279  0.43028605  1.84870269  77.3          1389
  30.00    82.00  0.18273117  2.40044694  0.37914027  2.35067062  0.33599701  2.29890345  0.47287972  1.97353155  78.2          1392

          PHI SUB A =  299.20 DEG, DIP =  66.90 DEG, HM = 0.510 GAUSS    DELTA Z= 0.10 LAMBDA

 FREQ     PHII       TEE AMPL   TEE PHASE    TEM AMPL    TEM PHASE   TME AMPL    TME PHASE    TMM AMPL    TMM PHASE HT          KLOCK
  30.00    65.00  0.30895406  0.42889414  0.22238814 -1.14200831  0.23888201 -1.40264562  0.43741572  1.63167652  79.1          1419
  30.00    73.00  0.32133278  1.42769882  0.33839888  0.77401499  0.36818328  0.75173612  0.32153268 -2.10332744  78.7          1404
  30.00    78.00  0.39863447  1.82595990  0.34873673  1.77533154  0.38771328  1.81292608  0.34801681 -0.33840968  79.0          1430
  30.00    80.00  0.45694583  1.99797248  0.33568951  2.15637183  0.37554602  2.20929661  0.40150820  0.32145615  79.2          1401
  30.00    81.00  0.49222184  2.09085258  0.32471278  2.34473475  0.36422063  2.40395604  0.43708498  0.63293077  79.3          1407
  30.00    82.00  0.53159478  2.18860970  0.31034911  2.53234626  0.34891375  2.59704933  0.47817761  0.93301088  79.4          1397
```

Figure 18. Printed output from ANIREF sample case, page 2.

ANIREF PUNCHED OUTPUT

```
                identification               azimuth    dip angle    field intensity
SAN FRAN SEAPATH   NIGHT       PHIA= 213.5 DIP=   6.5 HM= 0.310 GAUSS
  30.065.02.386-001 1.9271.830-001-1.0187.001-002 1.5413.332-001-1.049 74.0 76.3
  30.073.01.627-001 2.8252.093-001 0.9357.233-002-1.8514.838-001 0.565 74.0 74.9
  30.078.01.527-001 2.5762.187-001 1.8929.881-002-0.7885.981-001 1.402 74.0 76.2
  30.080.02.031-001 2.4752.135-001 2.2311.028-001-0.4436.503-001 1.713 74.0 77.1
  30.081.02.410-001 2.4752.081-001 2.3921.027-001-0.2816.781-001 1.865 74.0 77.4
  30.082.02.866-001 2.5042.005-001 2.5481.010-001-0.1257.073-001 2.014 74.0 77.6
SAN FRAN SEAPATH   NIGHT       PHIA= 258.1 DIP=  39.0 HM= 0.370 GAUSS
  30.065.03.816-001 1.1412.662-001-1.1152.343-001-0.8282.081-001 0.630 74.0 77.8
  30.073.02.596-001 2.6713.786-001 0.7353.404-001 0.7832.047-001 1.153 74.0 75.3
  30.078.01.394-001 2.9984.035-001 1.6713.596-001 1.6513.211-001 1.513 74.0 74.6
  30.080.01.270-001 2.6723.977-001 2.0173.533-001 1.9793.909-001 1.730 74.0 76.2
  30.081.01.464-001 2.4953.902-001 2.1853.462-001 2.1404.303-001 1.849 74.0 77.3
  30.082.01.827-001 2.4003.791-001 2.3513.360-001 2.2994.729-001 1.974 74.0 78.2
SAN FRAN SEAPATH   NIGHT       PHIA= 299.2 DIP=  66.9 HM= 0.510 GAUSS
  30.065.03.090-001 0.4292.224-001-1.1422.389-001-1.4034.374-001 1.632 74.0 79.1
  30.073.03.213-001 1.4283.384-001 0.7743.682-001 0.7523.215-001-2.103 74.0 78.7
  30.078.03.986-001 1.8263.487-001 1.7753.877-001 1.8133.480-001-0.338 74.0 79.0
  30.080.04.569-001 1.9983.357-001 2.1563.755-001 2.2094.015-001 0.321 74.0 79.2
  30.081.04.922-001 2.0913.247-001 2.3453.642-001 2.4044.371-001 0.633 74.0 79.3
  30.082.05.316-001 2.1893.103-001 2.5323.489-001 2.5974.782-001 0.933 74.0 79.4
```

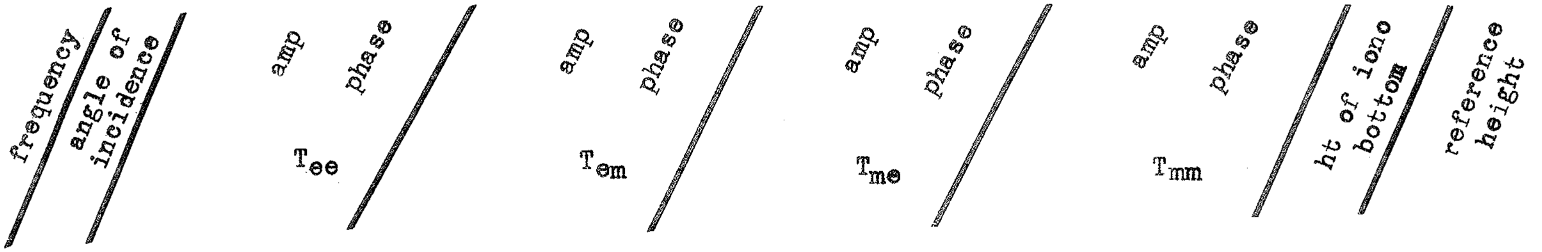


Figure 19. List of punched card output from ANIREF sample case.

The elements of the matrix are defined by Johler and Harper (1962) in their equations (33-35) (note an error in the general form of their subscripts). They depend on the roots of a quartic equation ((8) through (20) of Johler and Harper (1962)), which in turn depends on the ionospheric model, radio frequency, and direction of propagation.

Instead of solving (A2) directly, we divide the matrix into block matrices:

$$\begin{bmatrix} A_{11} & A_{12} & 0 & 0 & \cdot & \cdot & \cdot \\ A_{21} & A_{22} & A_{23} & 0 & \cdot & \cdot & \cdot \\ 0 & A_{32} & A_{33} & A_{34} & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & A_{p-1,p-2} & A_{p-1,p-1} & A_{p-1,p} \\ 0 & \cdot & \cdot & \cdot & 0 & A_{p,p-1} & A_{pp} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ \cdot \\ \cdot \\ U_p \end{bmatrix} = \begin{bmatrix} K \\ 0 \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix} \qquad \text{(A3)}$$

A_{ij} is 4×4, U_i and K are 4×2.

Consider the rows as separate equations and solve the last one for U_p

$$U_p = -A_{pp}^{-1} A_{p,p-1} U_{p-1} . \qquad \text{(A4)}$$

Substitute this value into the next-to-last equation, so it has only two unknowns and solve for U_{p-1}. Repeat the process until U_1 is found. The algorithm is: Let

$$B_p = A_{pp} , \qquad \text{(A5)}$$

and let

$$B_k = A_{kk} - A_{k,k+1} B_{k+1}^{-1} A_{k+1,k}, \text{ for } 1 \leq k < p . \qquad \text{(A6)}$$

Then

$$B_1 U_1 = K, \text{ or } U_1 = B_1^{-1} K \,. \tag{A7}$$

Since only columns 3 and 4 of $A_{k+1,k}$ and columns 1 and 2 of $A_{k,k+1}$ are nonzero, the elements of the A_{ij} are stored in a $4p \times 8$ array arranged as follows:

$$\begin{array}{rl} \text{Column No.} & \begin{array}{ccc} 1,2 & \quad 3,4,5,6 \quad & 7,8 \end{array} \\ & \left[\begin{array}{ccc} 0 & A_{11} & A_{12} \\ A_{21} & A_{22} & A_{23} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ A_{p,p-1} & A_{pp} & 0 \end{array}\right] \end{array} \tag{A8}$$

The real and imaginary parts of (A8) are stored in separate arrays. The rows a_{ik} and b_{ik} of each layer (see (A2)) have been interchanged for convenience. If the elements of (A8) are denoted α_{ij}, then

$$\alpha_{4p+1,j} = b_{p,m}, \; \alpha_{4p+2,j} = a_{p,m}, \tag{A9}$$

$$\alpha_{4p+3,j} = c_{p,m}, \text{ and } \alpha_{4p+4,j} = d_{p,m},$$

where $m = 4p + j - 6$, and a_{ij}, b_{ij}, c_{ij}, and d_{ij} are the elements of (A2).

After the array (A8) has been generated, the B_k are computed as follows:

(1) $B_k^{-1} A_{k,k-1}$ is computed and stored in A_{kk}. B_k^{-1} is found by solving $B_k B_k^{-1} = I$.

(2) Then B_{k-1} is calculated using (A6) and stored in $A_{k-1,k-1}$, and the process is repeated.

When B_1 has been calculated, columns 1 and 2 are interchanged with columns 3 and 4 (to prevent zeros on the diagonal in certain cases) and (A7) is solved. The reflection coefficients have now been found.

The phase of the reflection coefficient is referenced to the bottom of the ionospheric profile. We also compute the quasi-stationary phase height of T_{ee} as follows: There is a height where the phase of T_{ee} is nearly π over a considerable range of angles of incidence near grazing. In general the phase of T_{ee} at height h_r in terms of the phase at h_o is

$$p_r = p_o + \frac{4\pi}{\lambda} \cos \varphi (h_r - h_o) , \tag{A10}$$

where p_i is the phase at height h_i. So let $p_r = \pi$ and solve for h_r:

$$h_r = h_o + \frac{\pi - p_o}{\frac{4\pi \cos \varphi}{\lambda}} . \tag{A11}$$

We call h_r the quasi-stationary phase height.

7.4 FORTRAN Listing of Program ANIREF and Its Subroutines

PROGRAM ANI REF

```
C
C  COMPUTES IONOSPHERIC REFLECTION COEFFICIENTS FOR STRATIFIED IONOSPHERE
C     BY L. A. BERRY AND J. E. HERMAN, OFFICE OF TELECOMMUNICATIONS
C     REFERENCE IS TELECOMMUNICATIONS RESEARCH REPORT NO. 11   ,
C     ITS, BOULDER COLORADO, 80302
C
      DIMENSION FRE(9),SPHI(10),PHA(8),HA(8),DAP(8)
      DIMENSION PROFLE(5),EN(80),Z(80)
      DIMENSION PR(4),PI(4),QR(4),QI(4),ZR(4),ZI(4)
      DIMENSION AR(600,8), AI( 600,8)
      DIMENSION ROOT(8), COF(10)
      COMMON AR,AI,PR,PI,QR,QI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
     1 CI,SI,HM,OMEGA
     2  ,ZMA,WAVE,COF
      COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
     1 TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
    1 FORMAT(F10.2,E10.2)
    2 FORMAT(1H )
    3 FORMAT(*  FREQ    PHII      TEE AMPL   TEE PHASE    TEM AMPL    TE
     1M PHASE    TME AMPL    TME PHASE     TMM AMPL     TMM PHASE HT
     2   KLOCK *)
    4 FORMAT(1H1)
    6 FORMAT(2F8.2,8F12.8,F6.1,7X   ,I9)
    7 FORMAT(5A6,5H   H = F5.2,5H KM, ,I5,8H POINTS. )
    8 FORMAT(8F10.0)
    9 FORMAT(1H0,9X,12HPHI SUB A = F7.2,11H DEG, DIP = F7.2,
     110H DEG, HM = F6.3,18H GAUSS    DELTA Z=F5.2, 7H LAMBDA )
   19 FORMAT(I2)
   25 FORMAT(3F10.0)

C             RAT IS THE SLAB THICKNESS,  HERE = .1 WAVELENGTH

      DATA (RAT=.1)
   22 READ 19,IFR
C
C    IFR IS NUMBER OF FREQUENCIES
C
      IF(EOF,60)400,20
   20 READ 8,(FRE(I),I=1,IFR)
C
C             FRE IS THE FREQUENCY IN KHZ
      READ 19,NPA
C     NPA IS NUMBER OF MAGNETIC FIELD CONDITIONS
C
      READ 25,(PHA(I),HA(I),DAP(I),I=1,NPA)
C     PHA =  PHI A = MAGNETIC AZIMUTH
C     HA = HM  = MAGNETIC FIELD STRENGTH IN GAUSS
C     DAP = DIP = MAGNETIC FIELD DIP ANGLE IN DEGREES
C     CAUTION---(  (DIP=0) .AND.(PHIA=90.OR.270)  ) IS NOT ALLOWED
C
      READ 19,NPI
C      NPI = NUMBER OF ANGLES OF INCIDENCE, PHI I , IN DEGREES
C
```

```
      READ 8, (SPHI(I),I=1,NPI)
   30 READ 7,(PROFLE(I),I=1,5),HITE,LAYR
C
C  PROFLE  IS ALPHANUMERIC DESCRIBING IONO PROFILE, HITE IS HT OF BOTTOM OF
C  IONOS IN KM,  LAYR IS THE NUMBER OF DATA CARDS IN IONO PROFILE TO FOLLOW
C    A BLANK CARD RETURNS THE PROGRAM TO 22 TO READ A NEW SET OF PARAMETERS
C
      IF(EOF,60)400,31
   31 CONTINUE
      IF (LAYR) 22,22,32
   32 PRINT 4
      PRINT 4
      PRINT               7,(PROFLE(I),I=1,5),HITE,LAYR
      DO 33 L=1,LAYR
      READ 1,Z(L),EN(L)
C
C  Z(L) IS THE HEIGHT IN KM,  EN(L) IS THE ELECTRONS/CC AT HT Z(L)
C
      PRINT 1,Z(L),EN(L)
   33 CONTINUE
      PRINT 4
      PRINT               7,(PROFLE(I),I=1,5),HITE,LAYR
C
C       BEGINNING OF FREQUENCY LOOP
C
      DO 300  J=1,IFR
      F=FRE(J)
      DELTA = RAT*300./F
C
C       BEGINNING OF MAGNETIC PARAMETERS LOOP
C
      DO 290  K=1,NPA
      PHIA=PHA(K)  $  HM=HA(K)  $  DIP=DAP(K)
      PRINT 9,PHIA,DIP,HM,RAT
      LINE =0
      PRINT 2
      PRINT 2
      PRINT 3
      RDG=.017 453 292 519 943                                          MOD UP
      FIA=RDG*PHIA                                                      MOD UP
      DD= RDG*DIP                                                       MOD UP
      CFIA=COSF(FIA)                                                    MOD UP
      SFIA=SINF(FIA)                                                    MOD UP
      SI  =SINF(DD)
      CI  =COSF(DD)
  100 CONTINUE
   77 FORMAT(5A6,* PHIA=*,F6.1,* DIP=*,F6.1,* HM=*,F6.3,* GAUSS*)
      PUNCH 77, (PROFLE(I),I=1,5), PHIA,DIP,HM
C
C BEGINNING OF ANGLE OF INCIDENCE LOOP
C
      DO 280  L=1,NPI
      PHII=SPHI(L)
  110 FII=RDG*PHII                                                      MOD UP
```

```
      CFII=COSF(FII)                                                    MOD UP
      SFII=SINF(FII)                                                    MOD UP
      AT=SFII*SFIA                                                      MOD UP
      AL=SFII*CFIA                                                      MOD UP
      K1=KLOCK(0)
  150 CALL REFCOF
C
C    REFCOF COMPUTES THE REFLECTION COEFFICIENTS.  INPUT AND OUTPUT ARE IN
C  COMMON/1/
C
  160 K2=KLOCK(1)

C      COMPUTE QUASI STATIONARY PHASE HEIGHT

      HREF =HITE+(3.1416-TEEP)/(2.*WAVE*CFII)
      PRINT 6,F,PHII,TEEA,TEEP,TEMA,TEMP,TMEA,TMEP,TMMA,TMMP
     1,HREF    ,K2
      PUNCH 1111 , F,PHII,TEEA,TEEP,TEMA,TEMP,TMEA,TMEP,TMMA,TMMP,HITE,
     1 HREF
 1111 FORMAT(F6.1,F4.1,4(E9.3,F6.3),2F5.1)
  170 LINE=LINE+1
      IF(LINE.LE.50) GO TO 200
      PRINT 4
      PRINT               7,(PROFLE(I),I=1,5),HITE,LAYR
      PRINT 9,PHIA,DIP,HM,RAT
      LINE =0
      PRINT 2
      PRINT 3
      PRINT 2
  200 CONTINUE
C
C      END OF ANGLE OF INCIDENCE LOOP
C
  280 CONTINUE
  290 CONTINUE

C END OF FREQUENCY LOOP

  300 CONTINUE
      GO TO 30
  400 CALL EXIT
      END
```

```
      SUBROUTINE REFCOF
C
C     MAIN SUBROUTINE FOR ANIREF.  COMPUTES REFLECTION COEFFICIENTS.
C     REFERENCE IS TELECOMMUNICATIONS RESEARCH REPORT NO. 11   ,
C     BY L. A. BERRY AND J. E. HERMAN, OFFICE OF TELECOMMUNICATIONS
C     ITS, BOULDER COLORADO, 80302
C
      COMMON AR,AI,PR,PI,QR,QI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
     1 CI,SI,HM,OMEGA
     2   ,ZMA,WAVE,COF
      COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
     1 TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
      COMMON/4/ BAR,BAI
      DIMENSION PR(4), PI(4), QR(4), QI(4)
      DIMENSION BAR(4,8), BAI(4,8), COF(10), ROOT(8), P(8),Q(8)
      DIMENSION   AR(600,8), AI(600,8) ,Z(80), EN(80)
      OMEGA=6283.1853*F                                                 MOD UP
      WAVE=OMEGA/2.997925 E 5                                           MOD UP
      NLAY = (Z(LAYR) -Z(1)) / DELTA

C   THERE ARE NLAY SLABS

      IF( NLAY .LE. 149) GO TO 10
      NLAY = 149
      DELTA = (Z(LAYR) -Z(1)) / 149.
      DEL = WAVE *DELTA/6.2831853
      PRINT 9, DEL
    9 FORMAT ( *  CHANGED DELTA Z TO  *, F6.2, *  LAMBDA.*)
   10 CISA=CFII*SFIA                                                    MOD UP
      CDEL=3.1415927/(2.*WAVE)
      CICA=CFII*CFIA                                                    MOD UP
C
C   INITIAL GUESS FOR ROOTS OF BOOKER QUARTIC AT BOTTOM OF IONOSPHERE
C
      ROOT(1)=CFII
      ROOT(2)=0.
      ROOT(3)=CFII
      ROOT(4)=0.
      ROOT(5)=-CFII
      ROOT(6)=0.
      ROOT(7)=-CFII
      ROOT(8)=0.
      ZK=WAVE*DELTA
      ZM=Z(1)+.5*DELTA
      ELEC=ENN(ZM)                                                      MOD UP
      COL=CFEO(ZM)
C
C  ZM IS HEIGHT AT MIDDLE   OF SLAB
C  ELEC IS ELECTRON DENSITY OF SLAB, COL IS COLLISION FREQUENCY OF SLAB.
C
      CALL CQPZ(ELEC,COL)                                               MOD UP
      CALLFOLEST(8,1)

C         THE CALL TO Q9EXUN SURPRESSES THE SYSTEM EXPONENT UNDERFLOW ERROR
```

```
C          CHECK, DIAGNOSIS, AND HALT. (EXPONENT UNDERFLOWS ARE IGNORED)
      CALL Q9EXUN
C
C     FILL UP COEFFICIENT MATRIX.  SEE EQ(A2) AND (A8).
C
      DO 60 L=2,NLAY
      LP=4*L-3
      DO 40 J=1,4
      TX=ZK*ROOT(2*J)
   35 EX=EXPF(TX)
      TY=-ZK*ROOT(2*J-1)
      AR(LP,J)=-EX*COSF(TY)
      AI(LP,J)=-EX*SINF(TY)
      DO 40 K=1,3
      LPK=LP+K
      AR(LPK,J)=AR(LP,J)*AR(LPK-4,J+4)-AI(LP,J)*AI(LPK-4,J+4)
      AI(LPK,J)=AI(LP,J)*AR(LPK-4,J+4)+AR(LP,J)*AI(LPK-4,J+4)
      IF(AR(LPK,J)**2+AI(LPK,J)**2 .GE. 1.E-100) GO TO 40
      NLAY=L
   40 CONTINUE
      IF(L-NLAY)42,61,61
   42 ZM=ZM+DELTA
      ELEC=ENN(ZM)                                                      MOD UP
      COL=CFEO(ZM)                                                      MOD UP
      CALL CQPZ(ELEC,COL)                                               MOD UP
   60 CALLFOLEST(8,LP)
   61 HT = ZM + DELTA
      CALL CQPZ( ENN(HT), CFEO(HT))
      CALLFOLEST(6,LP)
      NTOP=NLAY-1
C
C     BEGIN RECURSIVE SOLUTION OF MATRIX EQ(A5),(A6).
C
      DO 150 L=1,NTOP
   71 LA=NLAY+1-L
      LP=4*LA-4
      DO 90 I=1,4
      LPI=LP+I
      DO 90 J=1,4
      BAR(I,J)=AR(LPI  ,J+2)
      BAI(I,J)=AI(LPI  ,J+2)
   90 CONTINUE
C      MINVERT INVERTS A 4X4 COMPLEX MATRIX
   98   CALL MINVERT
C
C     THIS LOOP IS STEP 1, FOLLOWING EQ(A9).
C
      DO 140 I=1,4
      LPA=LP+I
      DO 140 J=5,6
      JA=J-4
      AR(LPA,J)=BAR(I,5)*AR(LP+1,JA)-BAI(I,5)*AI(LP+1,JA)
      AI(LPA,J)=BAI(I,5)*AR(LP+1,JA)+BAR(I,5)*AI(LP+1,JA)
```

```
      DO 140 K=2,4
      LPK=LP+K
      TX=BAR(I,K+4)*AR(LPK,JA)-BAI(I,K+4)*AI(LPK,JA)
      TY=BAI(I,K+4)*AR(LPK,JA)+BAR(I,K+4)*AI(LPK,JA)
      AR(LPA,J)=AR(LPA,J)+TX
  140 AI(LPA,J)=AI(LPA,J)+TY
C
C     THIS LOOP IS STEP 2, FOLLOWING EQ(A9)
C
      DO 150 I=1,4
      LPA=LP-4+I
      DO 150 J=5,6
      JA=J+2
      TX=AR(LPA,7)*AR(LP+1,J)-AI(LPA,7)*AI(LP+1,J)
      TY=AI(LPA,7)*AR(LP+1,J)+AR(LPA,7)*AI(LP+1,J)
      SX=AR(LPA,8)*AR(LP+2,J)-AI(LPA,8)*AI(LP+2,J)
      SY=AI(LPA,8)*AR(LP+2,J)+AR(LPA,8)*AI(LP+2,J)
      AR(LPA,J)=AR(LPA,J)-TX-SX
  150 AI(LPA,J)=AI(LPA,J)-TY-SY
   91 DO 95 J=1,2
      DO 95 I=1,4
      BAR(I,J)=AR(I,J+4)
      BAI( I, J+2) =0.
   95 BAI(I,J)=AI(I,J+4)
      BAR(1,3)=SFIA
      BAR(1,4) = CICA
      BAR(2,3)=-CFIA
      BAR(2,4) = CISA
      BAR(3,3) = CISA
      BAR(3,4)=CFIA
      BAR(4,3) = CICA
      BAR(4,4)=-SFIA
C
C     SOLVE EQ (A7).
C
      CALL MINVERT
      TEMR = BAR(3,5)*CICA+BAR(3,6)*CISA-BAR(3,7)*CFIA+BAR(3,8)*SFIA
      TEMI = BAI(3,5)*CICA+BAI(3,6)*CISA-BAI(3,7)*CFIA+BAI(3,8)*SFIA
      TEER = BAR(4,5)*CICA+BAR(4,6)*CISA-BAR(4,7)*CFIA+BAR(4,8)*SFIA
      TEEI = BAI(4,5)*CICA+BAI(4,6)*CISA-BAI(4,7)*CFIA+BAI(4,8)*SFIA
      TMMR = -BAR(3,5)*SFIA + BAR(3,6) *CFIA+BAR(3,7)*CISA+BAR(3,8)*CICA
      TMMI = -BAI(3,5)*SFIA + BAI(3,6) *CFIA+BAI(3,7)*CISA+BAI(3,8)*CICA
      TMER = -BAR(4,5)*SFIA + BAR(4,6) *CFIA+BAR(4,7)*CISA+BAR(4,8)*CICA
      TMEI = -BAI(4,5)*SFIA + BAI(4,6) *CFIA+BAI(4,7)*CISA+BAI(4,8)*CICA
      TEEA=CABS(CMPLX(TEER,TEEI))
      TEEP=CANG(CMPLX(TEER,TEEI))
      TEMA=CABS(CMPLX(TEMR,TEMI))
      TEMP=CANG(CMPLX(TEMR,TEMI))
      TMEA=CABS(CMPLX(TMER,TMEI))
      TMEP=CANG(CMPLX(TMER,TMEI))
      TMMA=CABS(CMPLX(TMMR,TMMI))
      TMMP=CANG(CMPLX(TMMR,TMMI))
      RETURN                                                            MOD UP
      END
```

```
      FUNCTION ENN(ZM)
C
C  USES LOGARITHMIC INTERPOLATION IN TABLE OF N(H) TO GET ELECTRON DENSITY AT ZM

      DIMENSION Z(80),EN(80)
      COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
     1 TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
      KID=1
      IF(Z(LAYR)-ZM)70,70,25
   70 ENN=EN(LAYR)
      GO TO 71
   25 IF(Z(KID+1)-ZM)26,27,28
   26 KID=KID+1
      GO TO 25
   27 ENN=EN(KID+1)
      GO TO 71
   28 TX=(ZM-Z(KID))/(Z(KID+1)-Z(KID))
      ENN= EN(KID)*EXPF(TX*LOGF(EN(KID+1)/EN(KID)))
   71 CONTINUE
      RETURN
      END
```

```
      FUNCTION CFEO(H)
C
C     DASA COLLISION FREQUENCY MODEL FIT WITH EXP(P9),
C             WHERE P9 IS A NINTH DEGREE POLYNOMIAL.
C
      DIMENSION A(10)
      DATA((A(I),I=1,9 )=2.587803463E1, -1.210027715E-1, -1.462645167E-3
     1 , -1.172264046E-5, 1.749042668E-6, -2.948406644E-8, 1.351055095E-
     2 10, 4.11118378E-13, -3.289391577E-15 )
      SUM =A(9)
      DO 10 J=1,8
   10 SUM = H*SUM +A(9-J)
      CFEO = EXPF(SUM)
      RETURN
      END
```

```
      SUBROUTINE CQPZ(ELEC,COL)                                         MOD ONE
C
C  SUBROUTINE FOR REFCOF.  RETURNS THE ROOTS OF THE QUARTIC, AND THE
C     FIELD RATIOS P AND Q, SEE JOHLER AND HARPER, JAN 1961 RADIO SCIENCE.
C
      COMMON AR,AI,PR,PI,QR,QI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
     1 CI,SI,HM,OMEGA
     2  ,ZMA,WAVE,COF
      DIMENSION COF(10),PR(4),PI(4),QR(4),QI(4),ZR(4),ZI(4)
      DIMENSION AR(600,8), AI( 600,8)
      DIMENSION  ROOT(8),TOOT(4),ZOOTR(4),ZOOTI(4),ZZR(4),ZZI(4)
      JU=1
      JD=4
      Y=1.75888 E 7 *HM/OMEGA
      YT= Y*CI
      YL=-Y*SI
      X=(3.1824858 E 9*ELEC)/(OMEGA*OMEGA)
      Z=+COL/OMEGA
      ATSQ=AT*AT
      ALSQ=AL*AL
      CSQ =1.  -ATSQ-ALSQ
      XSQ=X*X
      YSQ =Y*Y
      ZSQ=Z*Z
      YLSQ=YL*YL
      YTSQ=YT*YT
      UR=1.
      UI=-Z
      U2R=1. -ZSQ
      U2I=-2. *Z
      TAR=1.-ZSQ-YSQ
      TAI=2.*Z
      TX=TAR*TAR+TAI*TAI
      TX=-X/TX
      TAR=TAR*TX
      TAI=TAI*TX
      PXXR=TAR   -TAI*UI
      PXXI=TAR*UI+TAI
      PXYR=-YL*TAI
      PXYI= YL*TAR
      PXZR= YT*TAI
      PXZI=-YT*TAR
      PZXR=-PXZR
      PZXI=-PXZI
      TX=1.+ZSQ
      TBR=(TAR-TAI*Z)/TX
      TBI=(TAI+TAR*Z)/TX
      TCR=U2R-YLSQ
      TCI=U2I
      PZZR=TBR*TCR-TBI*TCI
      PZZI=TBR*TCI+TBI*TCR
      PZYR=-YL*YT*TBR
      PZYI=-YL*YT*TBI
      TCR=U2R-YTSQ
```

```
      PYYR=TBR*TCR-TBI*TCI
      PYYI=TBR*TCI+TBI*TCR
      TCR=PXXR*(1.-ATSQ)+PYYR*(1.-ALSQ)+PZZR
      TCI=PXXI*(1.-ATSQ)+PYYI*(1.-ALSQ)+PZZI
C
C  COEFFICIENTS OF BOOKER QUARTIC
C
      COF(9)=1.+PZZR
      COF(10)=PZZI
      COF(7)=2.*AL*PZYR
      COF(8)=2.*AL*PZYI
      COF(5)=-2.*CSQ+2.*X*TAR-TCR-CSQ*PZZR
      COF(6)=        2.*X*TAI-TCI-CSQ*PZZI
      COF(3)=-CSQ*COF(7)
      COF(4)=-CSQ*COF(8)
      COF(1)=CSQ*CSQ+CSQ*TCR-(2.*CSQ+1.)*X*TAR+XSQ*TBR
      COF(2)=        CSQ*TCI-(2.*CSQ+1.)*X*TAI+XSQ*TBI
      CALL ZROOT
      DO 9 I=1,4
      II=I+I
      ZR(I)=ROOT(II-1)
      ZI(I)=ROOT(II)
    9 CONTINUE
      R12R=AL*AT+PXYR
      R12I=PXYI
      R33R=CSQ+PZZR
      R33I=PZZI
C
C   COMPUTE P AND Q IN THE 40 LOOP
C
      DO  40  I=1,4
      ZRSQ=ZR(I)*ZR(I)-ZI(I)*ZI(I)
      ZISQ=2. *ZR(I)*ZI(I)
      R11R=1.-ALSQ-ZRSQ+PXXR
      R11I=-ZISQ+PXXI
      R13R=AT*ZR(I)+PXZR
      R13I=AT*ZI(I)+PXZI
      R31R=AT*ZR(I)+PZXR
      R31I=AT*ZI(I)+PZXI
      R32R=AL*ZR(I)+PZYR
      R32I=AL*ZI(I)+PZYI
      DENR=R33R*R11R-R33I*R11I - R13R*R31R+R13I*R31I
      DENI=R13I*R31R+R13R*R31I - R33I*R11R-R33R*R11I
      TX=DENR*DENR +DENI*DENI
      PPR=R12R*R31R-R12I*R31I -R32R*R11R+R32I*R11I
      PPI=R12I*R31R+R12R*R31I -R32I*R11R-R32R*R11I
      QQR=R13R*R32R-R13I*R32I -R33R*R12R+R33I*R12I
      QQI=R13I*R32R+R13R*R32I -R33I*R12R-R33R*R12I
      TX1=PPR*DENR-PPI*DENI
      PPI=PPI*DENR+PPR*DENI
      PPR=TX1
      TX1=QQR*DENR-QQI*DENI
      QQI=QQI*DENR+QQR*DENI
      QQR=TX1
```

```
      PR(I)=PPR/TX
      PI(I)=PPI/TX
      QR(I)=QQR/TX
      QI(I)=QQI/TX
   40 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FOLEST(JJ,LP)
C
C     SUBROUTINE FOR REFCOF.  FILLS UP COL 5 - 8 OF EQ (A8).
C
      DIMENSION PR(4), PI(4), QR(4), QI(4), ROOT(8),P(8), Q(8)
      DIMENSION AR(600,8), AI( 600,8)
      DIMENSION COF(10)
      COMMON AR,AI,PR,PI,QR,QI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
     1 CI,SI,HM,OMEGA
     2  ,ZMA,WAVE,COF
      DO 100 K=1,4
      KK=2*K
      P(KK-1) = PR(K)
      P(KK) = PI(K)
      Q(KK-1) = QR(K)
  100 Q(KK) = QI(K)
      DO 20 J=5,JJ
      JP=2*J-9
      AR(LP,J)=1.
      AI(LP,J)=0.
      AR(LP+1,J)=Q(JP)
      AI(LP+1,J)=Q(JP+1)
      AR(LP+2,J)=AL*P(JP)-ROOT(JP)
      AI(LP+2,J)=AL*P(JP+1)-ROOT(JP+1)
      TX=ROOT(JP)*Q(JP)-ROOT(JP+1)*Q(JP+1)
      TY=ROOT(JP+1)*Q(JP)+ROOT(JP)*Q(JP+1)
      AR(LP+3,J)=TX-AT*P(JP)
   20 AI(LP+3,J)=TY-AT*P(JP+1)
      RETURN
      END
```

```
      SUBROUTINE ZROOT
C
C     ZROOT FINDS THE ROOTS OF A QUARTIC WITH COMPLEX COEFFICIENTS
C     USING THE NEWTON'RAPHSON METHOD TO FIND THE FIRST TWO ROOTS,
C     AND THE QUADRATIC FORMULA FOR THE OTHER TWO. COF(1)+I*COF(2) IS
C     THE CONSTANT TERM, COF(3)+I*COF(4) IS THE COEFFICIENT OF Z    ,ETC
C     THE ROOTS ARE ROOT(2*K-1)+I*ROOT(2*K),K=1,4 WRITTEN BY B. WEIDER
C
      DIMENSION COF(10),ROOT(8)
      DIMENSION AR(600,8), AI(600,8), VR(4), VI(4), WR(4), WI(4)
      COMMON AR,AI,VR,VI,WR,WI, ROOT, AL,AT,F,DELTA,CFII,SFIA,CFIA,
     1 CI,SI,HM,OMEGA
     2  ,ZMA,WAVE,COF
      J=1
      JRT=2
      JCOF=10
 1000 EPSILON=1.E-8
  999 JROOT=JRT+JRT
      L=0
C
C     USE ROOT FOR PREVIOUS CASE AS FIRST GUESS
C
      XR=ROOT(JROOT-1)
      XI=ROOT(JROOT)
    1 I=JCOF-J
      L=L+1
      YR=XR
      YI=XI
      FR=COF(I)
      FI=COF(I+1)
    2 I=I-2
      TX=XR*FR-XI*FI+COF(I)
      FI=XR*FI+XI*FR+COF(I+1)
      FR=TX
      IF(I.GT.1) GO TO 2
      A=FR
      B=FI
C
C     A+I*B=P(XR+I*XI)
C
      I=JCOF-J
      CCOF=(I-1)/2
      GR=CCOF*COF(I)
      GI=CCOF*COF(I+1)
    4 I=I-2
      CCOF=CCOF-1.
      TX=XR*GR-XI*GI+CCOF*COF(I)
      GI=XR*GI+XI*GR+CCOF*COF(I+1)
      GR=TX
      IF(I.GT.3) GO TO 4
      C=GR
      D=GI
C
C     C+I*D=DERIVATIVE OF P(XR+I*XI)
```

```
C
      ASSIGN 5 TO LSW
      GO TO 400
    5 EPSR=X
      EPSI=Y
C
C     EPSR+I*EPSI=CORRECTION TO ROOT.
C
      A=EPSR
      B=EPSI
      C=XR
      D=XI
      ASSIGN 6 TO LSW
      GO TO 400
    6 X=ABSF(X)
      Y=ABSF(Y)
C
C     HAVE WE CONVERGED
C
      IF(X.GT.EPSILON) GO TO 7
      IF(Y.LT.EPSILON) GO TO 60
    7 XR=XR-EPSR
      XI=XI-EPSI
      IF(L.LE.10) GO TO 1
      EPSILON=10.*EPSILON
      L=0
      IF(EPSILON .GT. 1.1E-6) PRINT 800,EPSILON,ZMA
  800 FORMAT(1H0,*  INCREASED EPSILON TO*,E9.1,* IN ZROOT AT HEIGHT*,
     1 F8.2//)
      GO TO 1
   60 ROOT(JROOT-1)=XR
      ROOT(JROOT)=XI
      N=JCOF-3
   61 A=COF(N+2)
      B=COF(N+3)
      X=A*XR-B*XI
      Y=A*XI+B*XR
      COF(N  )=COF(N  )+X
      COF(N+1)=COF(N+1)+Y
      N=N-2
      IF(N.GT.1)GO TO 61
      N=JCOF-2
      DO 62 K=1,N
      COF(K)=COF(K+2)
   62 CONTINUE
      JCOF=JCOF-2
      JRT=JRT+2
      IF(JRT.LE.4) GO TO 1000
C
C     FROM HERE TO  72+4 IS QUADRATIC FORMULA
C
      X=COF(3)*COF(3)-COF(4)*COF(4)
      A=COF(3)*COF(4)
      Y=A+A
```

```
      A=4.*(COF(1)*COF(5)-COF(2)*COF(6))
      B=4.*(COF(1)*COF(6)+COF(2)*COF(5))
      A=X-A
      B=Y-B
      ASSIGN 70 TO LSW
      GO TO 500
   70 A=X
      B=Y
      C=2.*COF(5)
      D=2.*COF(6)
      ASSIGN 71 TO LSW
      GO TO 400
   71 TX=X
      TY=Y
      A=COF(3)
      B=COF(4)
      ASSIGN 72 TO LSW
      GO TO 400
   72 A=-X+TX
      B=-Y+TY
      C=-X-TX
      D=-Y-TY
      IF(B) 73,75,74
   73 ROOT(1)=A
      ROOT(2)=B
      ROOT(5)=C
      ROOT(6)=D
      GO TO 78
   74 ROOT(1)=C
      ROOT(2)=D
      ROOT(5)=A
      ROOT(6)=B
      GO TO 78
   75 IF(A.GT.0.) GO TO 76
      ROOT(1)=C
      ROOT(5)=A
      GO TO 77
   76 ROOT(1)=A
      ROOT(5)=C
   77 ROOT(2)=0.
      ROOT(6)=0.
   78 CONTINUE
      RETURN
C
C     THIS IS A COMPLEX DIVISION BRANCH.  X+I*Y=(A+I*B)/(C+I*D)
C
  400 IF(C)410,405,410
  405 IF(D)406,440,406
  406 X=B/D
      Y=-A/D
      GO TO LSW
  410 IF(D)415,411,415
  411 X=A/C
      Y=B/C
```

```
      GO TO LSW
  415 IF(ABSF(C)-ABSF(D))416,430,430
  416 AX=C/D
      DEN=AX*C+D
      X =(A*AX+B)/DEN
      Y =(B*AX-A)/DEN
      GO TO LSW
  430 AX=D/C
      DEN=C+AX*D
      X=(A+B*AX)/DEN
      Y=(B-A*AX)/DEN
      GO TO LSW
  440 PRINT 401
  401 FORMAT(* ZERO DIVISOR  IN  ZMROOT DIVIDE*)
      CALL EXIT
C
C     THIS IS A COMPLEX SQUARE ROOT BRANCH. X+I*Y=SQRT(A+I*B).
C
  500 S=1.
      IF(B)501,505,505
  501 S=-1.
  505 IF(A)502,504,502
  502 X=SQRTF(.5*(SQRTF(A*A+B*B)+ABSF(A)))
      D=B/(2.*X)
      Y=D
      IF(A.GE.0.)GO TO LSW
      Y=S*X
      X=S*D
      GO TO LSW
  504 X=SQRTF(S*B*.5)
      Y=S*X
      GO TO LSW
      END
```

```
      SUBROUTINE MINVERT
C
C  INVERTS 4X4 COMPLEX MATRIX
C
      COMMON /4/ BR,AR,BI,AI
      DIMENSION IPVOT(4),INDEX(8),PIVOTR(4),PIVOTI(4)                    XIRTAM
      DIMENSION AR(16),AI(16),BR(16),BI(16)                              XIRTAM
      N=4   $  NSQ=16                                                    XIRTAM
      DO 888 I=1,NSQ
      AR(I)=BR(I)                                                        XIRTAM
      AI(I)=BI(I)                                                        XIRTAM
  888 CONTINUE
      DETR=1.                                                            XIRTAM
      DETI=0.                                                            XIRTAM
      DO 17 J=1,N
      IPVOT(J)=0                                                         XIRTAM
   17 CONTINUE
      DO 135 I=1,N
      TR=0.                                                              XIRTAM
      TI=0.                                                              XIRTAM
      DO 9 J=1,N
      IF(IPVOT(J).EQ.1)GO TO 9                                           XIRTAM
      NK=-N
      DO 23 K=1,N
      NK=NK+N
      JK=J+NK
      IF(IPVOT(K)-1)43,23,81                                             XIRTAM
   43 IF((TR*TR+TI*TI)-(AR(JK)*AR(JK)+AI(JK)*AI(JK)))83,23,23           XIRTAM
   83 IROW=J                                                             XIRTAM
      ICOL=K                                                             XIRTAM
      TR=AR(JK)                                                          XIRTAM
      TI=AI(JK)
   23 CONTINUE
    9 CONTINUE
      IPVOT(ICOL)=IPVOT(ICOL)+1                                          XIRTAM
      IF(IROW.EQ.ICOL)GO TO 109                                          XIRTAM
      DETR=-DETR                                                         XIRTAM
      DETI=-DETI                                                         XIRTAM
      NL=-N
      DO 12 L=1,N
      NL=NL+N                                                            XIRTAM
      IROWL=IROW+NL                                                      XIRTAM
      ICOLL=ICOL+NL                                                      XIRTAM
      TR=AR(IROWL)                                                       XIRTAM
      TI=AI(IROWL)                                                       XIRTAM
      AR(IROWL)=AR(ICOLL)                                                XIRTAM
      AI(IROWL)=AI(ICOLL)                                                XIRTAM
      AR(ICOLL)=TR                                                       XIRTAM
      AI(ICOLL)=TI                                                       XIRTAM
   12 CONTINUE
  109 I1=I                                                               XIRTAM
      I2=I+N                                                             XIRTAM
      ICOLICOL=ICOL+N*(ICOL-1)                                           XIRTAM
      INDEX(I1)=IROW                                                     XIRTAM
```

```
      INDEX(I2)=ICOL                                                    XIRTAM
      PIVOTR(I)=AR(ICOLICOL)                                            XIRTAM
      PIVOTI(I)=AI(ICOLICOL)                                            XIRTAM
      TXR=DETR*PIVOTR(I)-DETI*PIVOTI(I)                                 XIRTAM
      TXI=DETR*PIVOTI(I)+DETI*PIVOTR(I)                                 XIRTAM
      DETR=TXR                                                          XIRTAM
      DETI=TXI                                                          XIRTAM
      AR(ICOLICOL)=1.                                                   XIRTAM
      AI(ICOLICOL)=0.                                                   XIRTAM
      NL=-N
      DO 205 L=1,N
      NL=NL+N
      ICOLL=ICOL+NL                                                     XIRTAM
      DXR=PIVOTR(I)*PIVOTR(I)+PIVOTI(I)*PIVOTI(I)                       XIRTAM
      TXR=AR(ICOLL)*PIVOTR(I)+AI(ICOLL)*PIVOTI(I)                       XIRTAM
      TXI=AI(ICOLL)*PIVOTR(I)-AR(ICOLL)*PIVOTI(I)                       XIRTAM
      AR(ICOLL)=TXR/DXR                                                 XIRTAM
      AI(ICOLL)=TXI/DXR                                                 XIRTAM
  205 CONTINUE
      DO 135 LI=1,N
      IF(LI-ICOL)21,135,21                                              XIRTAM
   21 LIICOL=LI+N*(ICOL-1)                                              XIRTAM
      TR=AR(LIICOL)                                                     XIRTAM
      TI=AI(LIICOL)                                                     XIRTAM
      AR(LIICOL)=0.                                                     XIRTAM
      AI(LIICOL)=0.                                                     XIRTAM
      NL=-N                                                             XIRTAM
      DO 89 L=1,N
      NL=NL+N                                                           XIRTAM
      ICOLL=ICOL+NL                                                     XIRTAM
      LIL=LI+NL                                                         XIRTAM
      TXR=AR(ICOLL)*TR-AI(ICOLL)*TI                                     XIRTAM
      TXI=AI(ICOLL)*TR+AR(ICOLL)*TI                                     XIRTAM
      AR(LIL)=AR(LIL)-TXR                                               XIRTAM
      AI(LIL)=AI(LIL)-TXI                                               XIRTAM
   89 CONTINUE
  135 CONTINUE
      DO 3 I=1,N
      L1=N-I+1                                                          XIRTAM
      L2=L1+N                                                           XIRTAM
      IF(INDEX(L1)-INDEX(L2))19,3,19                                    XIRTAM
   19 JROW=INDEX(L1)                                                    XIRTAM
      JCOL=INDEX(L2)                                                    XIRTAM
      NJROW=N*(JROW-1)
      NJCOL=N*(JCOL-1)
      DO 549 K=1,N
      KJROW=K+NJROW
      KJCOL=K+NJCOL
      TR=AR(KJROW)                                                      XIRTAM
      TI=AI(KJROW)                                                      XIRTAM
      AR(KJROW)=AR(KJCOL)                                               XIRTAM
      AI(KJROW)=AI(KJCOL)                                               XIRTAM
      AR(KJCOL)=TR                                                      XIRTAM
      AI(KJCOL)=TI                                                      XIRTAM
```

```
  549 CONTINUE
    3 CONTINUE
   81 CONTINUE                                                  XIRTAM
      RETURN                                                    XIRTAM
      END
```

If no electron density-height function is known, use the subroutine. The electron density profile is a function of the sun's zenith angle, hence a function of month, day, and hour.

```
      FUNCTION ENN(H)
C                 MOD VIII OF THE D REGION
C       H IS HEIGHT IN KM, ENN IS ELECTRON DENSITY PER CC.
C      CHI IS SUNS ZENITH ANGLE IN DEGREES
C         USE ABSOLUTE VALUE OF CHI AS IT IS SYMMETRIC ABOUT ZERO
      DIMENSION Z(80),EN(80)
      COMMON/1/ Z,EN,LAYR,TEER,TEEI,TEEA,TEEP,TEMR,TEMI,TEMA,TEMP,
     1 TMER,TMEI,TMEA,TMEP, TMMR,TMMI,TMMA,TMMP
      DATA  (CHIO =0.), (EN1=80.), (H1=65.),(B=.15),(H2=72.),(SCALH=3.3)
      EQUIVALENCE (Z(2),CHI)
      IF (CHI .EQ. CHIO) GO TO 50
      CHIO =CHI
      H1=65.
      IF(CHI .GE. 90.3) H1=65.+(CHI-90.3)*1.03
      EN1=80.
      IF( CHI .GT. 97.) EN1=0.
      IF(CHI.LE.97.  .AND.  CHI.GT.95.) EN1=4.*(97.-CHI)
      IF(CHI.LE. 95.  .AND. CHI.GT.90.) EN1=80.*10.**((90.-CHI)/5.)
      B=.15
      IF( CHI .GE. 100. ) B=.35
      IF(CHI.GT.70. .AND. CHI.LT. 100.) B= .15+(CHI-70.)/30.*.20
      IF(CHI .GE. 90.) H2=94.
      IF(CHI .LT. 90.) H2=94.-22.*COS(CHI*.01745329252)
   50 Q=(H-H1)/SCALH
      IF(CHI.GT. 97.) 55,52
   52 CN=EN1*EXP(1.-Q-EXP(-Q))
      GO TO 60
   55 CN=0.
   60 DN=1000.*EXP(B*(H-H2))
      ENN=CN+DN
      RETURN
      END
```

Given the month, day, hour, latitude, and longitude, this subroutine calculates the sun's zenith angle which is used in the alternate Function ENN listed.

```
      FUNCTION SOL ZEN(MONTH,DAY,TIME,LAT,LONG)
CALCULATES THE SOLAR ZENITH  ANGLE AT THE GIVEN MONTH, DAY,TIME,LONG,LAT
C
C        MONTH, 1-12
C        DAY,    1-31
C        TIME,  HOUR= 0-24
C        LAT=LATITUDE, RADIANS,    +=NORTH,   -=SOUTH
C        LONG = LONGITUDE,  RADIANS  EAST

      REAL MONTH,LAT,LONG
      DATA (PI=3.141592654)
      GLAT=PI/180.*23.5*SIN(2.*PI*(30.5*(MONTH-3.)+DAY-21.)/365.)
      GLON =-PI/180.*15.*TIME+PI
      SOL ZEN=ACOS(SIN(LAT)*SIN(GLAT)+COS(LAT)*COS(GLAT)*COS(GLON-LONG))
      RETURN
      END
```